Class: XII - 2017-18

Subject : Mathematics

Mock Test Paper - II (SET- I)

SECTION-A

Question numbers 1 to 4 carry 1 mark each.

1. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A^{-1}) = (\det A)^k$.

2. Determine the value of the constant 'k' so that the function $f(x) = \begin{cases} \dfrac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$.

3. Evaluate: $\int_2^3 3^x \, dx$

4. If a line makes angles 90° and 60° respectively with the positive directions of x and y axes, find the angle which it makes with the positive direction of z-axis.

SECTION - B

Question numbers 5 to 12 carry 2 marks each.

5. Show that all the diagonal elements of a skew symmetric matrix are zero.

6. Find $\dfrac{dy}{dx}$ at $x = 1, y = \dfrac{\pi}{4}$ if $\sin^2 y + \cos xy = K$.

7. The volume of a sphere is increasing at the rate of 3 cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

8. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ is always increasing on $\mathbb{R}$.

9. Find the vector equation of the line passing through the point A (1, 2, –1) and parallel to the line $5x - 25 = 14 - 7y = 35z$.

10. Prove that if E and F are independent events, then the events E and F' are also independent.

11. A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. It takes one hour to make a bracelet and half an hour to make a necklace. The maximum number of hours available per day is 16. If the profit on a necklace is ₹100 and that on a bracelet is ₹300. Formulate an LPP. for finding how many of each should be produced daily to maximise the profit? It is being given that at least one of each must be produced.

12. Find: $\int \dfrac{dx}{x^2 + 4x + 8}$

SECTION–C

Question numbers 13 to 23 carry 4 marks each

13. Prove that $\tan\left\{\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}+\tan\left\{\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right\}=\frac{2b}{a}$.

14. Using properties of determinants, prove that $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$.

OR

Let $A=\begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}, B=\begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}, C=\begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$, find a matrix D such that $CD - AB = O$.

15. Differentiate the function $(\sin x)^x + \sin^{-1}\sqrt{x}$ with respect to x.

OR

If $x^m y^n = (x+y)^{m+n}$, prove that $\frac{d^2y}{dx^2} = 0$

16. Find: $\int \frac{2x}{(x^2+1)(x^2+2)^2}\,dx$

17. Evaluate: $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx$

OR

Evaluate: $\int_0^{3/2} |x \sin \pi x|\,dx$

18. Prove that $x^2 - y^2 = C(x^2+y^2)^2$ is the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)\,dy$, where C is a parameter.

19. Let $\vec{a} = \hat{i}+\hat{j}+\hat{k}, \vec{b} = \hat{i}$ and $\vec{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$ then

(*a*) Let $c_1 = 1$ and $c_2 = 2$, find c_3 which makes $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

(*b*) If $c_2 = -1$ and $c_3 = 1$, show that no value of c_1 can make $\vec{a}, \vec{b}$ and $\vec{c}$ coplanar.

20. If $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$. Also, find the angle which $\vec{a}+\vec{b}+\vec{c}$ makes with $\vec{a}$ or $\vec{b}$ or $\vec{c}$.

21. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X=0) = P(X=1) = p$ and $P(X=2) = P(X=3)$ such that $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$, find the value of p.

22. Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Do you also agree that the value of truthfulness leads to more respect in the society?

23. Solve the following L.P.P. graphically:

Minimise	$Z = 5x + 10y$
Subject to constraints	$x + 2y \le 120, x + y \ge 60, x - 2y \ge 0$
and	$x, y \ge 0$

SECTION–D

Question numbers 24 to 29 carry 6 marks each.

24. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations $x + 3z = 9, -x + 2y - 2z = 4,$ $2x - 3y + 4z = -3$

25. Consider $f: \mathbb{R}_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}-1}{3}\right)$

Hence find

(i) $f^{-1}(10)$

(ii) y if $f^{-1}(y) = \frac{4}{3}$

where $\mathbb{R}_+$ is the set of all non-negative real numbers.

OR

Discuss the commutativity and associativity of binary operation '*' defined on $A = Q - \{1\}$ by the rule $a * b = a - b + ab$ for all $a, b \in A$. Also find the identity element of * in A and hence find the invertible elements of A.

26. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

27. Using integration, find the area of region bounded by the triangle whose vertices are $(-2, 1)$, $(0, 4)$ and $(2, 3)$.

OR

Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration.

28. Solve the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.

29. Find the equation of the plane through the line of intersection of $\vec{r}.(2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and $\vec{r}.(\hat{i} - \hat{j}) + 4 = 0$ and perpendicular to the plane $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$. Hence find whether the plane thus obtained contains the line $x - 1 = 2y - 4 = 3z - 12$.

OR

Find the vector and Cartesian equations of a line passing through $(1, 2, -4)$ and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

SET–II (Uncommon Questions to SET–I)

12. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/sec, then find the rate of change of the slope of the curve when $x = 3$.

20. The random variable X can take only the values, 0, 1, 2, 3. Given that $P(2) = P(3) = p$ and $P(0) = 2P(1)$. If $\sum p_i x_i^2 = 2\sum p_i x_i$, find the value of p.

21. Using vectors find the area of triangle ABC with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

22. Solve the following L.P.P. graphically:

Maximise $Z = 4x + y$

Subject to following constraints $x + y \leq 50,$

$3x + y \leq 90,$

$x \geq 10$

$x, y \geq 0$

23. Find: $\int \frac{2x}{(x^2 + 1)(x^4 + 4)} dx$

28. A metal box with a square base and vertical sides is to contain 1024 cm^3. The material for the top and bottom costs ₹ 5 per cm^2 and the material for the sides costs ₹ 2.50 per cm^2. Find the least cost of the box.

29. If $A = \begin{pmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{pmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2;\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5;\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$

SET–III (Uncommon Questions to SET–I & SET–II)

12. If $y = \sin^{-1}(6x\sqrt{1 - 9x^2}), -\frac{1}{3\sqrt{2}} < x < \frac{1}{3\sqrt{2}}$, then find $\frac{dy}{dx}$.

20. Solve the following L.P.P. graphically:

Maximise $Z = 20x + 10y$

Subject to the following constraints $x + 2y \leq 28,$

$3x + y \leq 24,$

$x \geq 2,$

$x, y \geq 0$

21. Show that the family of curves for which $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$, is given by $x^2 - y^2 = cx$.

22. Find: $\int \frac{(3 \sin x - 2) \cos x}{13 - \cos^2 x - 7 \sin x} dx$

23. Solve the following equation for x:

$\cos(\tan^{-1} x) = \sin\left(\cot^{-1} \frac{3}{4}\right)$

28. If $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 2 \\ -3 & 1 & -1 \end{pmatrix}$, find A^{-1} and hence solve the system of equations $2x + y - 3z = 13,$ $3x + 2y + z = 4, x + 2y - z = 8.$

29. Find the particular solution of the differential equation

$\tan x \cdot \frac{dy}{dx} = 2x \tan x + x^2 - y; (\tan x \neq 0)$ given that $y = 0$ when $x = \frac{\pi}{2}$.

Pg 4/4.

Mock Test Paper-II
SET-I (Answers)

Section A

(Ans 1) $k = -1$ (Ans 2) $k = -3$ (Ans 3) $\frac{18}{\log_e 3}$ (Ans 4) $\theta = \pi/3, \frac{5\pi}{6}$

Section B

(6) $\frac{\pi}{4(\sqrt{2}-1)}$ (7) $3\ cm^2/s$ (9) $\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

(11) Max. $Z = 100x + 300y$
subject to constraints: $x + y \le 24$
$\frac{x}{2} + y \le 16$
$x \geq 1,\ y \geq 1$

(12) $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right) + C$

(12) OR → $\begin{bmatrix} -191 & -110 \\ 77 & 84 \end{bmatrix}$ (15)

(15) → $\frac{dy}{dx} = (\sin x)^x [x \cot x + \log \sin x] + \frac{1}{2\sqrt{x(1-x)}}$

(16) $\log\left|\frac{x^2+1}{x^2+2}\right| + \frac{1}{x^2+2} + C$

OR → $\frac{2}{\pi} + \frac{1}{\pi^2}$

(17) $\pi^2/4$

(19) (a) $c_3 = 2$ (20) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (21) $P = 3/8$

(22) $4/9$ (23) Min $Z = 300$ at $x = 60,\ y = 0$

(24) $x = 0,\ y = 5\ \&\ z = 3$ (25) (i) 1 (ii) 19 OR → Not commutative
Not associative
No identity element
No inverse element.

(27) 4 sq units (OR) → $\frac{4\pi}{3}$ sq units

(28) $y = \sin x$ (29) $\vec{r}\cdot(-5\hat{i} + 2\hat{j} + 12\hat{k}) = 47$.
Yes, the plane contains the given line

Uncommon Question - Set - II (Answers)

(12) -72 unit/s. (20) $P = 1/11$ (21) $\frac{\sqrt{274}}{2}$ sq units

(22) Maximum $Z = 120$ when $x = 30,\ y = 0$

(23) $\frac{1}{5}\log|x^2+1| - \frac{1}{10}\log|x^4+4| + \frac{1}{10}\tan^{-1}\frac{x^2}{2} + C$

(28) ₹1920.   (29) $x = 2,\ y = -3,\ z = 5$

Uncommon Questions - Set - III - Answers.

(12) $\frac{dy}{dx} = \frac{6}{\sqrt{1-9x^2}}$ (20) Maximum $Z = 200$ at $x = 4,\ y = 12$

(22) $10\log|\sin x - 4| - 7\log|\sin x - 3| + C$

(23) $x = 3/4$.

(28) $x = 1, y = 2, z = 3$ (29) $4y\sin x = 4x^2\sin x - \pi^2$