Class XII : 2017-18
Subject : Mathematics
Mock Test Paper - IV

SECTION–A

Question numbers 1 to 4 carry 1 mark each.

1. Find the cofactor of a_{12} in the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$.

2. The binary operation '*' $R \times R \to R$ is defined as $a * b = 2a + b$. Find $2 * (3 * 4)$.

3. Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

4. Find the direction ratios of the line passing through the two points (–2, 4, –5) and (1, 2, 3).

SECTION–B

Question numbers 5 to 12 carry 2 marks each.

5. If $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, find the value of $3A - B$.

6. Differentiate w.r.t. x

$$\sin^{-1}\left(\frac{5x + 12\sqrt{1-x^2}}{13}\right)$$

7. Find the slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$.

8. For the set $A = \{1, 2, 3\}$, define a relation R in the set A as follows:
$R: \{(1, 1), (2, 2), (3, 3), (1, 3)\}$. Write the ordered pairs to be added to R to make it smallest equivalence relation.

9. Find the direction cosines of the line passing through the point $P(2, 3, 5)$ and $Q(-1, 2, 4)$.

10. Write integrating factor of $\cos x \frac{dy}{dx} + y \sin x = 1$.

11. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes two hour on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹ 5 and that from a shade is ₹ 3. Assuming that the manufacturer can sell all the lamps and shades that he produced, formulate the above problems in LPP to maximise his profit.

12. Two cards are drawn at random one by one without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

SECTION–C

Question numbers 13 to 23 carry 4 marks each.

13. Prove that for $a \geq 1, f(x) = \sqrt{3}\sin x - \cos x - 2ax + 4$ is decreasing in R.

OR

Find two positive numbers whose sum is 15 and the sum of whose cubes is minimum.

14. Using properties of determinants show that :

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1 - x^3)^2$$

15. Show that: $\int_0^{\frac{\pi}{2}} \sqrt{\tan x} + \sqrt{\cot x}\, dx = 2\pi$

OR

Evaluate: $\int x^2 \tan^{-1} x\, dx$

16. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'(x)$. Also find $f'\left(\frac{\pi}{2}\right)$.

OR

If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, find $\frac{dy}{dx}$.

17. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

18. Solve the differential equation

$$x \cos y dy = (xe^x \log x + e^x)dx.$$

19. Find the equation of the plane that passes through the points (2, 1, 0), (3, – 2, – 2) and (3, 1, 7).

20. If A, B, C, D are the points with position vectors $\hat{i} + \hat{j} - \hat{k}, 2\hat{i} - \hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{k}, 3\hat{i} - 2\hat{j} + \hat{k}$ respectively, find the projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$.

21. A toy company manufactures two types of dolls A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand per doll of type B is at most half of that for dolls of type A. Further the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of ₹ 12 and ₹ 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximize the profit?

22. *In a group of 100 families, 50 families like male child, 30 families like female child and 20 families feel both children are equal. If two families are selected at random out of 100 families, find the probability distribution of the number of families who feel both children are equal.*
What is the importance of developing the feeling that both children are equal in the society?

23. An insurance company insured 3000 cyclists, 4000 scooterists and 5000 car drivers. The probabilities of the accident involving a cyclists, scooterists and car drivers are 0.02, 0.03 and 0.04 respectively. One of the insured vehicle drivers meets with an accident. Find the probability that he is a car driver.

SECTION–D

Question numbers 24 to 29 carry 6 marks each.

24. Show that the exponential function $f: R \to R$, given by $f(x) = e^x$ is one-one but not onto. What will happen if the co-domain is replaced by R^+ (set of all positive real numbers)?

25. Find the matrix P satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

OR

If $\sum \cos^2 \alpha_1 = \sum \cos^2 \beta_1 = \sum \cos^2 \gamma_1 = 1$ and

$\sum \cos\alpha_1 \cos\beta_1 = \sum \cos\beta_1 \cos\gamma_1 = \sum \cos\gamma_1 \cos\alpha_1 = 0$

Then, find the value of

$$\begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \cos\alpha_3 \\ \cos\beta_1 & \cos\beta_2 & \cos\beta_3 \\ \cos\gamma_1 & \cos\gamma_2 & \cos\gamma_3 \end{vmatrix}$$

26. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a cone of height h is $\frac{1}{3}h$.

27. Find: $\int_0^{\pi/2} \sin 2x . \tan^{-1}(\sin x) dx$

OR

Evaluate: $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$

28. If $x = \sin t$ and $y = \sin pt$, prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2 y = 0$$

29. Find the coordinates of the foot of perpendicular drawn from the point A (1, 8, 4) to the line joining the points B (0, – 1, 3) and C (2, – 3, – 1).

ANSWERS

1. 46
2. 14
3. 1
4. 3, – 2, 8
5. $\begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$
6. $\frac{1}{\sqrt{1-x^2}}$
7. $-\frac{1}{3}$
8. (3, 1)
9. $\frac{3}{\sqrt{11}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{11}}$
10. $\sec x$
11. $Z = 5x + 3y$ which is to be maximised under constraints.

$3x + 2y \le 20$

$2x + y \le 12$

$x, y \ge 0$

where x, y are the number of pedestal lamps and wooden shades respectively and Z is profit.

12. $\frac{25}{102}$

13. OR $\frac{15}{2}, \frac{15}{2}$

15. OR $\frac{x^3}{3}\tan^{-1}x - \frac{x^2}{6} + \frac{1}{6}\log|x^2+1| + C$

16. $f'(x) = -\text{cosec}\, x.\cot x + \text{cosec}^2 x, f'\left(\frac{\pi}{2}\right) = 1$ OR $\frac{dy}{dx} = \frac{2xy + 2x - y^2}{2xy + 2y - x^2}$

17. πab sq. unit

18. $\sin y = e^x \log x + C$ [Hint: $\cos y dy = e^x\left(\log x + \frac{1}{x}\right)dx$]

19. $7x + 3y - z - 17 = 0$

20. $\sqrt{21}$

21. Type A dolls = 800

Type B dolls = 400

22.

X	0	1	2
P(X)	$\frac{632}{990}$	$\frac{320}{990}$	$\frac{38}{990}$

To maintain the male and female ratio, it is necessary to develop the feeling that both male and female child are equal.

23. $\frac{10}{19}$

24. When co-domain is replaced by R^+ function becomes one-one.

25. $P = \begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$ OR 1

27. $\frac{\pi}{2} - 1$ OR $\frac{-2}{\sin\alpha}\sqrt{\frac{\sin(x+\alpha)}{\sin x}} + C$

29. $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$

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