

Class XII : 2017-18
Subject : Mathematics
Mock Test Paper - V

SECTION-A

Question numbers 1 to 4 carry 1 mark each.

1. The binary operation $*$: $R \times R \rightarrow R$ is defined as $a*b = 2a + b$. Find $(3*2)*5$.
2. Find the value of $\tan^{-1} \left[\sin \left(-\frac{\pi}{2} \right) \right]$.
3. Find the value of x if $\begin{vmatrix} x & 4 \\ 2 & 2x \end{vmatrix} = 0$.
4. Find the vector equation of the line which is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$ and which passes through the point $(1, -2, 3)$.

SECTION-B

Question numbers 5 to 12 carry 2 marks each.

5. Let $*$ be a binary operation on $N \times N$, by $(a, b) * (c, d) = (a + c, b + d)$.
Show that $*$ is commutative and associative.
6. If $A = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, then for what value of α , A is an identity matrix?
7. If $f(x) = |\cos x - \sin x|$, find $f' \left(\frac{\pi}{6} \right)$.
8. Prove that the function $f(x) = \tan x - x$ is always increasing.
9. Solve the following linear programming problem graphically:
Maximise $Z = 4x + y$ subject to the constraints
 $x + y \leq 50$
 $3x + y \leq 90$
 $x \geq 0, y \geq 0$
10. Name the curve for which the slope of the tangent at any point is equal to the ratio of the abscissa to the ordinate of the point.
11. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.
12. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then find $P(A' \cap B')$.

SECTION-C

Question numbers 13 to 23 carry 4 marks each.

13) Using properties of determinant prove that:

$$\begin{vmatrix} x & p & q \\ p & x & q \\ q & q & x \end{vmatrix} = (x-p)(x^2 + px - 2q^2)$$

14. Show that the function $f(x)$ given by

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x & x \neq 0 \\ 2 & x = 0 \end{cases}$$

is continuous at $x = 0$.

OR

If $\sin y = x \sin(a+y)$ prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

15. Using integration find the area of a triangle ABC whose vertices have coordinates $A(2, 5)$, $B(4, 7)$ and $C(6, 2)$.

16. An aeroplane can carry a maximum of 200 passengers. A profit of ₹400 is made on each first class ticket and a profit of ₹300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximise profit for the airline. Form an LPP and solve it graphically.

17. Find the value of a for which the curve $y = x^2 + ax + 25$ touches the x -axis.

18. Evaluate: $\int \frac{2x}{(x^2+1)(x^2+3)} dx$

OR

Evaluate: $\int \frac{1 + \sin 2x}{1 + \cos 2x} e^{2x} dx$

19. Solve the differential equation

$$xe^{y/x} - y \sin\left(\frac{y}{x}\right) + x \frac{dy}{dx} \cdot \sin\left(\frac{y}{x}\right) = 0, \text{ when } y(1) = 0.$$

OR

Solve: $\frac{dy}{dx} = \sin(x+y)$

20. Prove that for any three vectors $\vec{a}, \vec{b}, \vec{c}$;

$$[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}] = 2[\vec{a} \quad \vec{b} \quad \vec{c}]$$

21. Find the equations of the two lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at angles of $\frac{\pi}{3}$ each.

22. In a village, there are 100 people, out of them 70 people are non-vegetarian. Two people are selected randomly. Find the probability distribution of vegetarian people. Which type of people is better? Give your opinion, keeping in mind the importance of life of animal in eco-system.

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23. A bag contains 5 red, 6 white and 7 black balls. Two balls are drawn at random. What is the probability that both balls are red or both are black?

SECTION-D

Question numbers 24 to 29 carry 6 marks each.

24. If $A = R - \{0\}$ and $*$ defined on A such that $a * b = 2ab$ then prove that

- (i) $*$ is commutative
 (ii) $*$ is associative
 (iii) write identity element of $*$ on A
 (iv) If inverse exists find the inverse of 'a'.

25. Evaluate:
$$\begin{vmatrix} \cos \alpha \cos \beta & \cos \alpha \sin \beta & -\sin \alpha \\ -\sin \beta & \cos \beta & 0 \\ \sin \alpha \cdot \cos \beta & \sin \alpha \cdot \sin \beta & \cos \alpha \end{vmatrix}$$

OR

Solve the system of linear equation, using matrix method:

$$x - y + z = 4$$

$$2x + y - 3z = 0$$

$$x + y + z = 2$$

26. Evaluate:
$$\int_0^{\pi} \frac{x \, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$$

OR

Evaluate:
$$\int \frac{\sin 2x \cdot \cos 2x}{\sqrt{9 - \cos^4 2x}} \, dx$$

27. Differentiate the following function $f(x)$ w.r.t. x , where $f(x) = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$.

28. A monument is standing vertically on earth which behaves like a plane having equation $2x - 3y + 4z + 9 = 0$. If the peak of monument lies at a point having coordinate $(1, -2, 3)$, then find the height of the monument.

OR

Find the equation of the plane passing through the point $(-1, -1, 2)$ and perpendicular to each of the following planes:

$$2x + 3y - 3z = 2 \text{ and } 5x - 4y + z = 6$$

29. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semivertical angle α is one-third that of the cone and the greatest volume of cylinder is $\frac{4}{27} \pi h^3 \tan^2 \alpha$.

ANSWERS

1. 21

2. $-\frac{\pi}{4}$

3. $x = \pm 2$

4. $\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 6\hat{k})$

6. $\alpha = 0$

7. $-\frac{1}{2}(1 + \sqrt{3})$

9. Max. value of $Z = 120$ at $x = 30, y = 0$

10. Rectangular hyperbola

11. $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

12. $\frac{2}{9}$

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15. 7 sq units 16. Maximum profits ₹ 64,000, 40 tickets of first class and 160 of second class

17. $a = \pm 10$ 18. $\frac{1}{2} \log \left| \frac{x^2+1}{x^2+3} \right| + C$ OR $e^{2x}(\tan x) + C$

19. $e^{-y/x} \{ \sin y / x + \cos y / x \} = 1 + \log x^2, x \neq 0$

OR

$$\tan(x+y) - \sec(x+y) = x + C$$

[Hint: Put $x+y=v$ and obtain $\frac{dv}{1+\sin v} = dx$]

21. $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x}{-1} = \frac{y}{1} = \frac{z}{-2}$

22.

X	0	1	2
P(X)	$\frac{483}{990}$	$\frac{420}{990}$	$\frac{87}{990}$

Vegetarian is better than non-vegetarian as food habit of non-vegetarian will decrease the number of some animals and thus it will cause bad effect on environment.

23. $\frac{31}{153}$

24. (iii) Identity element = $\frac{1}{2}$; (iv) Inverse of $a = \frac{1}{4a}$

25. 1 OR $x=2, y=-1, z=1$

26. $\frac{\pi^2}{2ab}$ OR $-\frac{1}{4} \sin^{-1} \left[\frac{1}{3} \cos^2 2x \right] + C$

27. $\frac{1}{2} \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \left[\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5} \right]$

28. $\sqrt{29}$ units OR $9x + 17y + 23z = 20$