Class - X

Question Bank (Mathematics) (2017-18)

Very Short Questions

* Question numbers from 1 to 30 carry 1 mark each.

1. In a family of 3 children, find the probability of having atleast one boy. (Ans $\frac{7}{8}$)

2. A card is drawn at random from a well shuffled pack of 52 cards. Find the probability of getting neither a red card and nor a queen. (Ans : $\frac{1}{13}$)

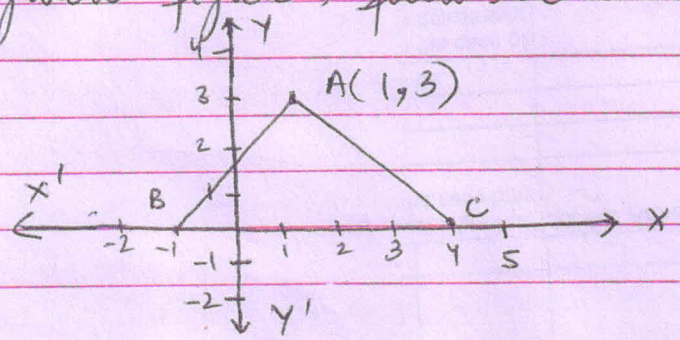
3. What is the empirical relationship among the median, mode and mean of a data?

4. What is the median class of the following data? (Ans 20-30)

| Class | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | Total |
|---|---|---|---|---|---|---|
| Frequency | 12 | 8 | 8 | 15 | 3 | 46 |

5. The mid point of the line segment AB is the point P(0,4). If the coordinates of B are (-2,3), then what are the coordinates of A? (Ans 3,5)

6. In the given figure, find the area of $\triangle ABC$. (Ans 7 sq units)

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7. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet. Find x : y. Ans (1 : 3)

8. The tops of two poles, of heights 25 m and 35 m are connected by a wire which makes an angle of elevation of 30° at the top of 25 m pole. Find the length of the wire. (Ans 20 m)

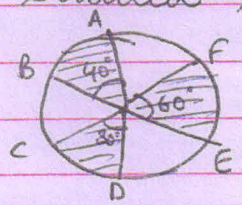
9. If $\cot A + \frac{1}{\cot A} = 1$, then find the value of $\cot^2 A + \frac{1}{\cot^2 A}$. (Ans −1)

10. If $\sqrt{3}\sin\theta = \cos\theta$, find the value of $\frac{\sin\theta \cdot \tan\theta(1+\cot\theta)}{\sin\theta + \cos\theta}$. (Ans $\frac{1}{\sqrt{3}}$)

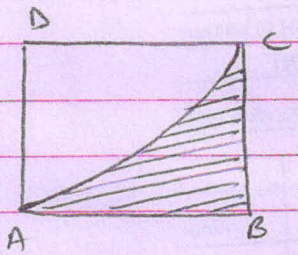
11. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to its base. If its volume is $\frac{1}{27}$ of the volume of the given cone, then find the height above the base at which the section is made. (Ans 20 cm)

12. A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm partly filled with water. If the sphere is completely submerged, then how much does the water level rise? (Ans 3cm)

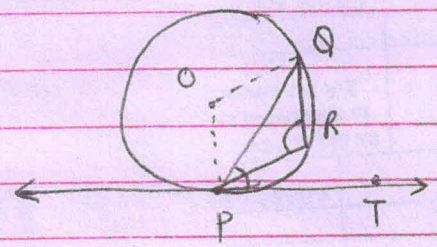
13. In the given figure, the radius of the circle with centre O is 7 cm, $\angle POQ = 60^\circ$, $\angle AOB = 40^\circ$ and $\angle COD = 80^\circ$. Find the area of shaded region. (Use $\pi = \frac{22}{7}$).
Ans 77 cm².



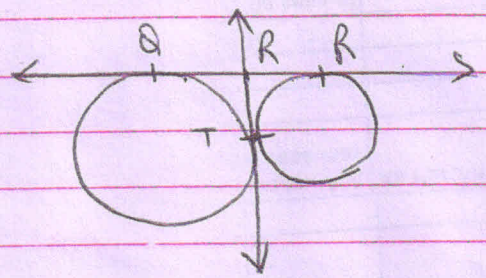

14. ABCD is a square of side 10 cm. What is the area of the shaded region where $\pi = 3.142$, correct to 1 decimal place.
(Ans 21.5 cm²)




15. In the adjoining figure, PQ is a chord of a circle with centre O. Line PT is a tangent. If $\angle QPT = 60^\circ$, find $\angle PRQ$.
(Ans 120°)




16. In the given figure, QR is a common tangent to the given circles, touching externally at the point T. The tangent at T meets QR at P. If PT = 3.8 cm, find the length of QR (in cm)

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17. In an isosceles $\Delta ABC$, if $AC = BC$ and $AB^2 = 2AC^2$, then find $\angle C$ (Ans $90°$)

18. ABCD is a trapezium in which $BC \parallel AD$. If $AB = 4$cm and the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$, then find BC. (Ans 4.6cm)

19. Find the $9^{th}$ terms from the end of the AP : 5, 9, 13, ..... 185. (Ans 153)

20. Which is the first negative term of the sequence $20, \frac{77}{4}, \frac{37}{2}, 7\frac{1}{4}$...? (Ans $28^{th}$)

21. Find the roots of the equation $a^2b^2x^2 + (b^2 - a^2)x - 1 = 0$ (Ans $-\frac{1}{a^2}, \frac{1}{b^2}$)

22. Find the value(s) of k for which the quadratic equation $kx^2 + 5x + k = 0$ has real and equal roots.

23. The pair of linear equations $2x + 3y = 11$ and $(m+n)x + (2m-n)y = 33$ has infinitely many solutions. Find the values of m and n. (Ans $m = 5, n = 1$)

24. Solve : (Ans : $x = -1, y = -1$)
$3x + 4y + 7 = 0$ and
$5x - 7y - 2 = 0$

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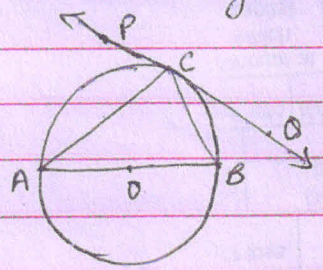
25. Find the value of p for which the polynomial $x^3+4x^2-px+8$ is exactly divisible by $(x-2)$. (Ans 16)

26. If $\alpha$ and $\beta$ are the zeroes of the polynomial $2x^2+5x+1$, find the value of $\alpha+\beta+\alpha\beta$. (Ans $-2$)

27. After how many places will the decimal expansion of $\frac{147}{120}$ terminate? (Ans 3)

28. Find the largest number which divides 70 and 125 leaving remainders 5 and 8 respectively. (Ans 13)

Q-29. PQ is a tangent to a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30^\circ$, find $\angle PCA$.




[Ans: $60^\circ$]

Q-30 If $\sin\theta = \cos\theta$, then find the value of $2\tan\theta + \cos^2\theta$.

[Ans: $\frac{5}{2}$]

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## Short Answer Question 2 Marks Questions

Q-1 → Find the least number of square tiles required to pave the ceiling of a room 15 m 17 cm long and 9 m 2 cm broad. [Ans: 814]

Q-2 → If the HCF of 210 and 55 is expressible in the form $210 \times 5 + 55y$. Find y. [Ans: $y = -19$]

Q-3 → If one zero of the polynomial $(a^2+9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of a. [Ans: $a = 3$]

Q-4 → If the sum of the zeroes of the polynomial $p(x) = (a+1)x^2 + (2a+3)x + (3a+4)$ is $-1$, then find the product of its zeroes. [Ans: 2]

Q-5 → A person, rowing at the rate of 5 km/hr in still water, takes thrice as much time in going 40 km upstream as on going 40 km downstream. Find the speed of the stream. [Ans: 5/2 km/hr]

Q-6 → Find the value of $\alpha$ and $\beta$ for which the following pair of linear equations has infinite no. of solutions

$2x + 3y = 7$ , $2\alpha x + (\alpha+\beta)y = 28$ [Ans $\alpha = 4, \beta = 12$]

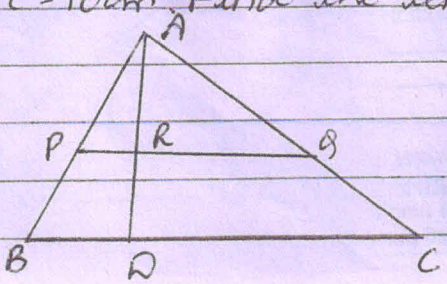
Q-7 → Solve: $x = \dfrac{1}{2 - \dfrac{1}{2 - \dfrac{1}{2-x}}}$ , $x \neq 2$ [Ans: $x = 1, 1$]

Q-8 → Solve the following quadratic equation by factorisation method: $\dfrac{x+3}{x-2} - \dfrac{(1-x)}{x} = \dfrac{17}{4}$ [Ans $x = -2/9$]

Q-9 → Find the sum of all odd integers between 2 and 100 which are divisible by 3. [Ans 867]

Q-10 → Find the sum of first 15 terms of an A.P. whose $n^{th}$ term is $3 - 2n$. [Ans $S_{15} = -195$]

Q-11 → In the given figure, AP = 3 cm, AR = 4·5 cm, AQ = 6 cm, AB = 5 cm, AC = 10 cm. Find the length of AD (Ans = 7·5 cm)

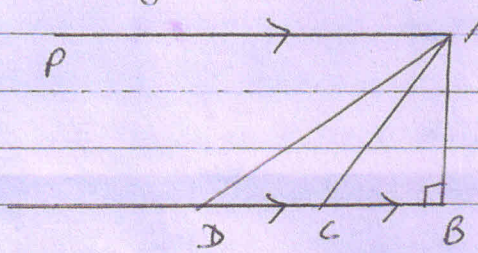


Q-12 → ABC is a right angled triangle at C. If p is the length of the perpendicular from C to AB and AB = c, BC = a, CA = b, then prove that $pc = ab$ and $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Q-13 → A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, -5) is the mid-point of PQ, then find the coordinates of P and Q. [Ans: (0, -10) (4, 0)]

Q-14 → Find the area of the triangle whose sides are along the lines $x = 2$, $y = 0$ and $4x + 5y = 20$ [Ans: $\frac{18}{5}$ sq. units]

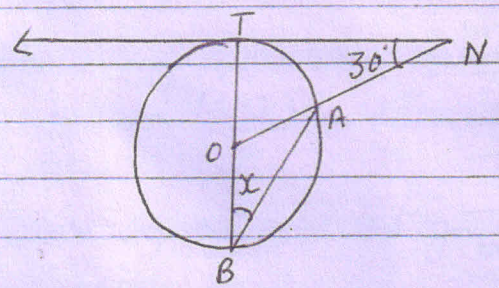
Q-15 → In the given figure, AB = 10 cm, ∠PAD = 45° and ∠PAC = 60°, find the length of CD.




[Ans: $\left(\frac{10\sqrt{3} - 10}{\sqrt{3}}\right)$ m]

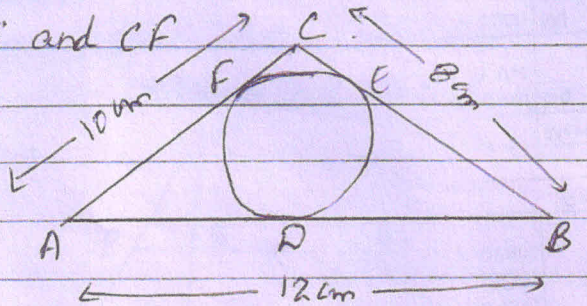
Q-16 → Find the sun's altitude when the height of the tower is $\sqrt{3}$ times of the length of its shadow [Ans: 60°]

Q-17 → Calculate the value of x, if TN is tangent to given figure



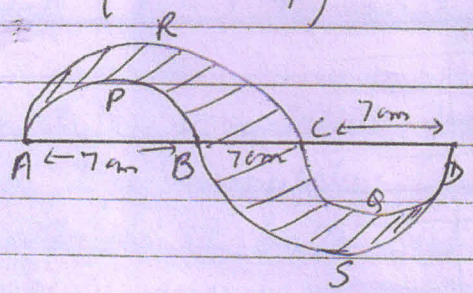

[Ans: 30°]

Q-18 → A circle is inscribed in a ΔABC having sides 8 cm, 10 cm and 12 cm as shown in the figure. Find AD, BE and CF




[Ans: 7 cm, 5 cm, 3 cm]

Q-19 → In the given figure, APB and CQD are semi-circles of diameter 7 cm each, while ARC and BSD are semicircles of diameter 14 cm each. Find the perimeter of the shaded region $\left(\text{Use } \pi = \frac{22}{7}\right)$




[Ans: 66 cm]

Q-20 → The height of a cone is 5 m. Find the height of another cone sixteen times its volume and radius equal to its diameter [Ans: 20 cm]

Q-21 → The altitude AD of a $\triangle ABC$, in which $\angle A$ is an obtuse angle has length 10 cm. If BD = 10 cm and CD = $10\sqrt{3}$ cm determine $\angle A$. [Ans: $\angle A = 105°$]

Q-22 → If $x = a\cos\theta$, $y = b\sin\theta$, then find the value of $b^2x^2 + a^2y^2 - a^2b^2$ [Ans: 0]

Q-23 → A sphere of maximum volume is cut-out from a solid hemisphere of radius 6 cm. What is the volume of cut-out sphere? [Ans: $36\pi$ cm$^3$]

Q-24 → If the area of a sector of a circle is 5/18 th of the area of that circle, then find the central angle of the circle. [Ans: 100°]

Q-25 → Find the value of p, if the mean of the following distribution is 27

| Classes | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 |
|---|---|---|---|---|---|
| Frequency | 8 | p | 12 | 13 | 10 |

[Ans: p = 7]

Q-26 → The following are the ages of 300 patients getting medical treatment in a hospital on a particular day:

| Age (in yrs.) | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 |
|---|---|---|---|---|---|---|
| No. of patients | 60 | 42 | 55 | 70 | 53 | 20 |

Form:

(i) Less than type cumulative frequency distribution

(ii) More than type cumulative frequency distribution

Q-27 → A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing black ball is now double of what it was before. Find x.

[Ans: $\frac{x}{12}$, $x = 3$]

## LONG ANSWER TYPE (3 Marks/4 marks)



Q1. Show that only one of the numbers $n, n+2, n+4$ is divisible by 3

Q2. Show that any positive odd integer is of the form $4q+1$ or $4q+3$, where $q$ is a positive integer.

Q3. Can we have any $n \in N$, where $7^n$ ends with the digit zero?

Q4 Find the HCF and LCM of 288, 360 and 384 by prime factorisation method. Ans (24, 5760)

Q5 Prove that $\sqrt{17}$ is irrational. Write two rational Nos b/w $\sqrt{2}$ and $\sqrt{3}$.

Q6 Prove that $15+17\sqrt{3}$ be an irrational number.

Q7 $\alpha, \beta$ are zeroes of the polynomial $x^2-6x+a$, find the value of $a$, if $3\alpha+2\beta=20$.

Q8 On dividing $x^3+x^2+x-2$ by a polynomial $g(x)$, the quotient and remainder were $x^2+2x+1$ and $2x-1$ respectively. Find $g(x)$.

Q9 Given that $x-\sqrt{5}$ is factor of the polynomial $x^3-3\sqrt{5}x^2-5x+15\sqrt{5}$, find all the zeroes of the polynomial.

Q10 If $x+a$ is a factor of the polynomial $x^2+px+q$ and $x^2+mx+n$ Prove that $a = \frac{n-q}{m-p}$.

Q11 Solve the equations graphically $2x+y=2$ and $2y-x=4$. What is the area of the triangle formed by the two lines and line $y=0$? Ans (5 sq units)

Q12 Solve the following pair of linear equations for $x$ and $y$:-
$2(ax-by)+(a+4b)=0$ and $2(bx+ay)+(b-4a)=0$

Q13 A takes 3 hours more than B to walk 30 km. But if A doubles his pace, he is ahead of B by 1½ hours. Find their speed?

Q14 A railway half ticket cost half the full fare and the reservation charge is the same on half ticket as on full ticket. One reservation first class ticket from Mumbai to Ahmedabad cost ₹216 and one full and one half reserved first class tickets cost ₹327. What is the basic first class full fare and what is the reservation charge?

Q15 A boat goes 24 km upstream and 28 km downstream in 6 hrs. It goes 30 km upstream and 21 km down stream in $6\frac{1}{2}$ hrs. Find the speed of boat in still water and also speed of stream.

Q16 Using quadratic formula, Solve the following quadratic equation for x:-

(i) $p^2x^2 + (p^2 - q^2)x - q^2 = 0$ (ii) $ax^2 + cx + a = 0$

Q17 Solve the following for x:- $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$.

Q18 The sum of the Areas of two squares is $640\ m^2$. If the difference in their perimeters be 64 m find the sides of the two squares.

Q19 A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more then the first speed. If it takes 3 hrs to Complete the total journey, what is the first speed?

Q20 If the roots of the equation

$(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are equal. Prove that- either $a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

Q21 An AP Consists of 21 terms. The sum of the three terms in the middle is 129 and of the last three is 237. find AP?

Q22 If $S_1, S_2, S_3$ are the sum of n terms of three AP's, the first term of each being unity and the respective common difference being 1, 2, 3. Prove that- $S_1 + S_3 = 2S_2$.

Q23 If the $p^{th}$ term of AP is $\frac{1}{q}$ and $q^{th}$ term is $\frac{1}{p}$, show that the sum of pq terms is $\frac{1}{2}(pq+1)$.

Q24 Show that the sum of an AP whose first term is a, the second term b and the last term c, is equal to $\frac{(a+c)(b+c-2a)}{2(b-a)}$.

Q25 If M, N and T are in AP, Prove that- $(M+2N-T)(2N+T-M)(T+M-N) = 4MNT$.

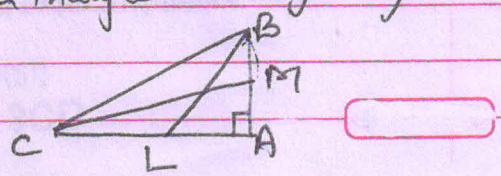
Q26 In an equilateral triangle ABC, D is a point on side BC such that $4BD = BC$. Prove that $16AD^2 = 13BC^2$.

Q27 If A is the Area of a right angled triangle and b is one of the sides containing right angle. Prove that the length of altitude on the hypotenuse is $\frac{2Ab}{\sqrt{b^4+4A^2}}$.

Q28 If the angles of one triangle are respectively equal to another triangle, Prove that the ratio of the corresponding sides is the same as the ratio of the corresponding:-
(i) medians (ii) bisectors of angles (iii) altitudes.

Q29 In an equilateral triangle PQR, the side QR is trisected as S. Prove that $9PS^2 = 7PQ^2$.

Q30 In fig, BL and CM are medians of a triangle ABC right angled at A. Prove that $4(BL^2+CM^2) = 5BC^2$.



Q31 In $\Delta PAB$, $PA = PB$ and area of $\Delta PAB = 10$ sq units. Find the coordinates of P if A and B are $(1,2)$ and $(3,8)$ respectively.

Q32 Prove that $(-2,3)$, $(8,3)$ and $(6,7)$ are the vertices of a right angled triangle.

Q33 The points $A(2,6)$, $B(9,1)$, $C(11,6)$ and $D(4,4)$ are the vertices of a quadrilateral ABCD. Determine whether ABCD is rhombus or not.

Q34 Prove the following identity $\frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta} = \sec\theta\operatorname{cosec}\theta + \cot\theta$.

Q35 Prove that $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$.

Q36 Prove that $\sec^2 x - \left[\frac{\sin^2 x - 2\sin^4 x}{2\cos^4 x - \cos^2 x}\right] = 1$

Q37 If $a\cos\theta - b\sin\theta = c$, prove that $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$.

Q38 If $\frac{\sin\theta+\cos\theta}{\sin\theta-\cos\theta} = \frac{5}{3}$ find the value of $\frac{7\tan\theta+2}{2\tan\theta+7}$.

Q39 In an acute angled triangle ABC, if $\sin(A+B-C) = \frac{1}{2}$ and $\cos(B+C-A) = \frac{1}{\sqrt{2}}$ find $\angle A$, $\angle B$ and $\angle C$.

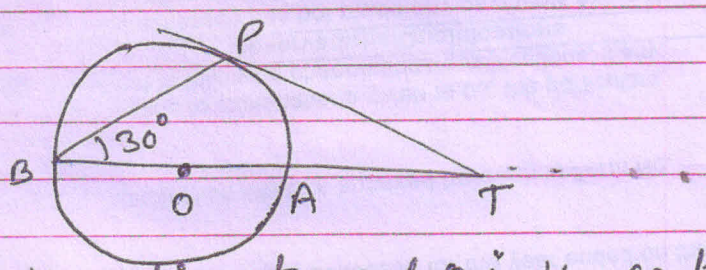
Q40 If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$ Prove that $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$.

Q41 (a) The angle of elevation of a cloud from a point 60m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake.

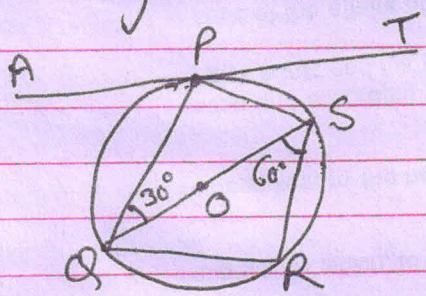
Q41 (b) If $\sec\theta + \tan\theta = p$ find the value of $\operatorname{cosec}\theta$.

Q42 At the foot of mountain, the elevation of its summit is $45^\circ$. After ascending 1000 m towards the mountain up a slope of $30^\circ$ inclination, the elevation is found to be $60^\circ$. Find the height of the mountain.

Q43 In figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If $\angle PBT = 30^\circ$, Prove that BA : AT = 2 : 1.

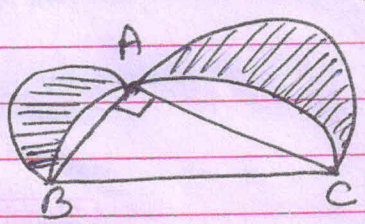



Q44 In the figure, QS is the diameter and O is the centre of circle. APT is the tangent at P. Find $\angle APQ$.




Q45 Draw a triangle ABC with BC = 7 cm, $\angle B = 45^\circ$ and $\angle A = 105^\circ$, then Construction of another $\Delta$ whose sides are $\frac{4}{5}$ times the corresponding sides of $\Delta ABC$.

Q46 In fig, ABC is a right-angled triangle, right-angled at A. Semicircle are drawn on AB, AC and BC as diameters. Find the area of the shaded region. Ans 6 sq units



Q47 A bucket has top and bottom diameter of 40cm and 20 cm respectively. Find the volume of the bucket if its depth is 12cm. Also find the cost of tin sheet used for making the bucket at the rate of ₹1.20 per $dm^2$.

Ans ($8800cm^3$, $1787.02cm^2$)
Cost = ₹21.44

Q48 Water is flowing at the rate of 2.52 km/h through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40cm. If the increase in the level of water in the tank, in half an hour is 3.15m, find the internal diameter of the pipe.

Q49 Find the mean, median and mode of the following data:-

| Classes | 0-20 | 20-40 | 40-60 | 60-80 | 80-100 | 100-120 | 120-140 |
|---|---|---|---|---|---|---|---|
| Frequency | 6 | 8 | 10 | 12 | 6 | 5 | 3 |

Q50 The median of the following data is 52.5. Find the values of x and y, If the total frequency is 100.

| CI | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 | 80-90 | 90-100 |
|---|---|---|---|---|---|---|---|---|---|---|
| f | 2 | 5 | x | 12 | 17 | 20 | y | 9 | 7 | 4 |

Q51 Two dice are numbered 1,2,3,4,5,6 and 1,2,2,3,3,4 respectively. They are thrown and the sum of the numbers on them is noted. Find the probability of getting (i) sum 7 (ii) Sum is a perfect square.

Ans $\left(\frac{1}{6}, \frac{7}{36}\right)$.

Q52 A letter is chosen at random from the letters of the word "ASSASSINATION" find the probability letter is vowel?