Revision Worksheet - XII - Mathematics - 2017-18

Topic : Differential Equations

1) If $\frac{dy}{dx} = \frac{xy+y}{xy+x}$, find the solution of the differential eqn. (Ans: $y = Axe^{x-y}$)

2) Verify if $y = 2x-4$ is a solution of the D.E. $(y_1)^2 - xy_1 + y = 0$.

3) Find a, b s.t. $y = ae^{-1/x} + b$ is a solution of the D.E. $\frac{dy}{dx} = \frac{y}{x^2}$ (Ans: $a \in R$, $b = 0$)

4) The solution curves of the D.E. $x\,dx - dy = 0$ are given by a family of: (A) parabolas (B) hyperbola (C) circles (D) ellipses (Tick the appropriate option)

5) Form the D.E. corresponding to the curve:
$y = A\sin(\omega t + \omega_0) + B\cos(\omega t + \omega_0)$; ω, ω_0 are known constants.

6) Find the order & degree of the given D.E:
(i) $y^2 = 2c(x + \sqrt{c})$ where c is a positive parameter
(ii) $y = c_1e^x + c_2e^{2x} + c_3e^{3x} + c_4e^{x+c_5}$; c_1, c_2, c_3, c_4, c_5 are arbitrary constants

7) The slope of the curve at any point is the reciprocal of twice the ordinate at that point. The curve passes through (3, 4). Find the equation of the curve.

8) Find the equation of the curve which is such that the portion of the x-axis cut off between the origin & the tangent at any point is proportional to the ordinate of that point. (Ans: $x = y(c - k\log y)$)

9) A curve has the property that if the tangent drawn at any point P on the curve meets the co-ordinate axes at A & B, then P is mid point of AB. The curve passes through (1,1). Find the equation of the curve (Ans: $xy = 1$)

10) Form the D.E corresponding to the curves:
(i) $y^2 = 4a(x+a)$ (ii) $y = ae^{bx}$

11) The slope of the tangent at (x, y) to a curve passing through $(1, \frac{\pi}{4})$ is given by: $\frac{y}{x} - \cos^2\left(\frac{y}{x}\right)$. Find the equation of the curve.

12) The rate of increase of bacteria in a certain culture is proportional to the number present. If it doubles in 5 hours, then in 25 hours its number would be how many times the original number (Ans: 32 times)

13) Solve the given differential equations :

(i) $\frac{dy}{dx} = 2y + 3e^x$; $y(0) = 0$ (ii) $(x + \log y)dy + y\,dx = 0$.

(iii) $\frac{dy}{dx} = \sin^2(x+3y) + 5$ (iv) $x^2 \frac{dy}{dx} = \frac{y(x+y)}{2}$

(v) $\frac{dy}{dx} = \cos(x-y)$ (vi) $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

(vi) $(2x - 10y^3)dy + y\,dx = 0$ (vii) $x\frac{dy}{dx} = y(\log y - \log x + 1)$

(vii) $\left(x \sin \frac{y}{x}\right) dy = \left(y \sin \frac{y}{x} - x\right) dx$ (viii) $(x+y)\frac{dy}{dx} = 4x + y$.

(ix) $\log\left(\frac{dy}{dx}\right) = 4x - 2y - 2$; $y(1) = 1$

(x) $\left(y - x\frac{dy}{dx}\right) = 3\left(1 - x^2 \frac{dy}{dx}\right)$ (xi) $xy\frac{dy}{dx} = \frac{1+y^2}{1+x^2}(1 + x + x^2)$

(xi) $x\frac{dy}{dx} = y + 2\sqrt{y^2 - x^2}$ (xii) $(1 - 2e^{x/y})dx - 2e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$

(xiii) $\frac{dy}{dx} = \frac{3x - 4y - 2}{3x - 4y - 3}$ (xiv) $x\frac{dy}{dx} = x\cot\frac{y}{x} + y$.

(xv) $(x^2+1)y' + 2xy = 4x^2$ (xvi) $(1-x^2)\frac{dy}{dx} + 2xy = x(1-x^2)^{1/2}$