QUESTION BANK
Class - XII
Subject - MATHEMATICS

Type – I (ONE MARK QUESTIONS)

1. If A = $\begin{pmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{pmatrix}$ find x, $0 < x < \pi/2$ when A + A' = I.

2. If B is a skew symmetric matrix, write whether the matrix (ABA') is symmetric or skew symmetric.

3. If A = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ show that A-A' is skew symmetric where A^T denotes transpose of A.

4. If A is a skew symmetric matrix of order 3 × 3 s.t. $a_{12} = 2$, $a_{13} = 3$ & $a_{23} = 5$, find matrix A.

5. If A is a skew symmetric matrix of order 3 × 3, find |A|.

6. If $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix, find x.

7. Construct the matrix $A = [aij]_{2\times3}$ where aij = 2i - 3j

8. If A is a square matrix of order 3 s.t. |AdjA| = 64, find |A| & $|A^{-1}|$.

9. If A = $\begin{bmatrix} 8 & -5 \\ 3 & -2 \end{bmatrix}$, find A^{-1}.

10. Find x if [x - 5 -1 $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

11. If $\begin{bmatrix} 2x+1 & 2y \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & y+5 \\ 0 & 26 \end{bmatrix}$, find x & y.

12. Using determinants find the area of the triangle with vertices (4, 4), (3, -2) & (-3, 16).

13. Using determinants find the equation of the straight line joining the points (1,2) & (3, 6).

14. Find the cofactor of a_{13} in $\begin{vmatrix} 3 & 0 & 5 \\ 4 & -2 & 4 \\ 1 & 3 & 2 \end{vmatrix}$

15. Give an example of a 3 × 3 scalar matrix.

16. If A_{ij} is the cofactor of a_{ij} of a 3×3 matrix then find the value of $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$.

17. Find the point on the curve $y = x^2 - 2x + 3$ where the tangent is || to the x-axis.

18. If $f(1) = 4$, $f^1(1) = 2$ find the value of the derivative of $\log f(e^x)$ w.r.t x at the point $x = 0$.

19. Find a for which $f(x) = a(x + \sin x) + a$ is increasing.

20. If normal to the curve at a point Pon $y = f(x)$ is parallel to the y-axis, what is the value of $\frac{dy}{dx}$ at P.

21. Discuss the applicability of Rolle's theorem for

    i. $f(x) = |x|$ on $[-1, 1]$

    ii. $f(x) = \frac{x(x-2)}{x-1}$ on $[0, 2]$

    iii. $f(x) = 2 + (x - 1)^{2/3}$ on $[0, 2]$

22. Give an example of a function which is continuous at all points but not differentiable at three points.

23. Differentiate $\tan^{-1}\left(\frac{a\sin x + b\cos x}{a\cos x - b\sin x}\right)$ w.r.t. x

24. If $x = at^2$, $y = 2$ at find $\frac{d^2y}{dx^2}$ at $t = -1$.

25. If $A = [-1\ -2\ -3]$ find AA' where A' is transpose of A.

26. Evaluate : $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$

27. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, find k if $|2A| = k\,|A|$

28. Find the least value of a.s.t. $f(x) = x^2 + ax + 1$ is strictly increasing on $(1, 2)$

29. Find the points on the curve $y = x^3$ at which slope of tangent is equal to the y-coordinate.

30. Evaluate $\int_0^{\sqrt{2}} \sqrt{2 - x^2}\, dx$

31. Evaluate $\int_{-4}^{-1} \frac{dx}{x}$

32. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx$

33. Evaluate $\int \frac{dx}{\sqrt{x+4} - \sqrt{x-2}}$

34. Evaluate $\int_0^{\pi/2} \log\left(\frac{3+5\cos x}{3+5\sin x}\right) dx$

35. Evaluate where $\int_0^{1.5} [x] dx$ where [x] represents the greatest integer function.

36. Evaluate $\int e^{3\log x} x^4 \, dx$

37. Write the order and degree of the D.E.

i) $y = x\frac{dy}{dx} + a\sqrt{1 + \frac{d^2y}{dx^2}}$

ii) $\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$

38. Solve the D.E.

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

39. Find a unit vector along $\vec{a} + \vec{b}$ if $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ & $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$

40. Find the vector in the direction of the vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.

41. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

42. Find $|\vec{a} - \vec{b}|$ if vectors $\vec{a}$ & $\vec{b}$ are s.t. $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} . \vec{b} = 4$

43. If $\vec{a}$ & $\vec{b}$ are two unit vectors and θ is the angle between them, show that $\sin \theta/2 = \frac{1}{2}|\hat{a} - \hat{b}|$

44. If $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$ find the angle between $\vec{a}$ and $\vec{b}$

45. Consider two points P & Q with P.V. $\vec{OP} = 3\vec{a} - 2\vec{b}$ & $\vec{OQ} = \vec{a} + \vec{b}$. Find the P.V. of
R which divides the line joining P & Q in the ratio 2:1 externally.

47. Evaluate :

$$\hat{i}\cdot(\hat{j}\times\hat{k})+\hat{j}\cdot(\hat{k}\times\hat{i})+\hat{k}\cdot(\hat{j}\times\hat{i})$$

48. If $\vec{a}=\hat{i}+\hat{j}$, $\vec{b}=\hat{j}+\hat{k}$, $\vec{c}=\hat{k}+\hat{i}$, find a unit vector in the direction of $\vec{a}+\vec{b}+\vec{c}$

49. Find the angle between the vectors $\hat{i}-2j+3\hat{k}$ & $3\hat{i}-2\hat{j}+\hat{k}$

50. If $\vec{a}$ is a unit vector and $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a}) = 8$ find $|\vec{x}|$

51. Find the direction cosines of the vector $2\hat{i}+\hat{j}-2\hat{k}$

52. Find x s.t. $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector

53. Find λ s.t. $2\hat{i}+3j-k$ & $-4\hat{j}-6\hat{j}+\lambda\hat{k}$ are

(i) Parallel
(ii) Perpendicular

55. Mention the points of discontinuity of the following functions :

(i) f(x) = [x]

(ii) $f(x) = \dfrac{x^2}{x^2-3x+2}$

56. Find the absolute maximum and absolute minimum value of :

$f(x) = 2\cos x + x \;;\; x \in [0, \pi]$

57. Find the slope of the tangent to the curve $y=\dfrac{x-7}{(x-2)(x-3)}$ at the point where it cuts the x-axis.

58. At what point on the curve y = x^2 does the tangent make an angle of 45^{o} with the x-axis?

59. Find the slope of normal to the curve $x=\dfrac{1}{t},\; y=2t \;\; at \;\; t=2$.

60. Evaluate :

i) $\displaystyle\int_{-1}^{1} e^{|x|}\,dx$

ii) $\displaystyle\int_{-\pi/2}^{\pi/2} \sin^7 x\,dx$

iii) $\displaystyle\int_{0}^{\pi/2} \frac{\sin x-\cos x}{1+\sin x\cos x}\,dx$

iv) $\displaystyle\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right)dx$

v) $\int_0^1 \frac{dx}{\sqrt{2x+3}}$ vi) $\int_0^{1/\sqrt{2}} \frac{dx}{\sqrt{1-x^2}}$

61. If $\int_0^1 (3x^2 + 2x + k)\, dx = 0$, find the value of k.

62. Evaluate $\int \frac{e^x (1+x)}{\cos^2 (xe^x)}\, dx$

63. Find the general solution of the differential equation :

$$\frac{ydx - xdy}{y} = 0$$

64. Find the order and degree of the following differential equation :

i) $\frac{d^4 y}{dx^4} - \sin\left(\frac{d^3 y}{dx^3}\right) = 0$

ii) $\frac{d^2 y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^{1/2}}$

65. What is the no. of arbitrary constants in the particular solution of a differential equation of order 3?

66. Find the integrating factor of the differential equation :

$$x \log x \frac{dy}{dx} + y = 2 \log x$$

67. Find a unit vector parallel to the sum of the vectors :

$-2\hat{i} + 2\hat{j} - 3\hat{k}$ & $\hat{i} - \hat{j} - \hat{k}$

68. Find and $\vec{a} \cdot \vec{b}$ if $|\vec{a}| = 2, |\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$

69. If $\vec{a}$ is any vector in space show that :
$\vec{a} = (\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$

70. If $2\hat{i} + \lambda\hat{j} + \hat{k}$ & $-5\hat{i} + 3\hat{j} + \mu\hat{k}$ are collinear, find λ, μ

71. If a vector makes angles : α, β, γ with the x, y, z - axis respectively, find the value of :
$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

72. Find the angle between the following pair of lines :

$\vec{r} = (\hat{i} + 5\hat{j} + 3\hat{k}) + \lambda\,(2\hat{i} - \hat{j} + 6\hat{k})$

and $\vec{r} = (3\hat{i} - \hat{j} + 2\hat{k}) + \mu\,(\hat{i} + 5\hat{k})$

73. Find the equation of the plane passing through the point (2, -1, 3) and parallel to the plane x + 2y + 4z = 20.

74. Find the intercepts cut off by the plane 4x + 5y - 2z = 10 on the y, z axes.

75. Find the vector equation of the line passing through the point (3, 4, 1) and parallel to the line $\vec{r} = (\hat{i} - 7\hat{j} + \hat{k}) + \lambda\,(8\hat{i} + j + \hat{k})$

76. Find the distance of the point (2, 3, 4) from the plane $\vec{r} = (3\hat{i} - 6\hat{j} + 2\hat{k}) = -11$

77. The Cartesian equation of a line AB is:
$\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction ratios of a line parallel to AB.

78. Find the vector equation of the plane which is at a distance of 5 units from the origin and is perpendicular to the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$

79. Find the angle between the line $\frac{x+2}{4} = \frac{y-1}{-5} = \frac{z}{7}$ & the plane 3x - 2z + 4 = 0

80. A line in the xy plane makes angle $\frac{\pi}{6}$ with the x-axis. Find the direction ratios and direction cosines of the line.

81. Reduce the equation $\vec{r}\,.\,(4\hat{i} - 6\hat{j} + 12\hat{k}) = 5$ to the normal form and hence find the length of perpendicular from the origin to the plane.

82. Find the equation of the plane which passes through the point (2, -3, 1) & is perpendicular to the line through the points (3, 4, -1) & (2, -1, -5).

83. If 4x + 4y – kz = 0 is the equation of the plane through the origin that contains the line $\frac{x-1}{2} = \frac{y+1}{3} = z$ find the value of K.

84. If a * b = $a^2 + b^2$, find the value of (1 * 2) * 3

85. Let * be a binary operation on N given by :

    a * b = L.C.M. of a & b. Find the value of 20 * 16.

86. If * defined on Z+ as :

    a * b = a – b, a binary operation ? Justify your answer.

87. Check whether the binary operation * defined on Q as follows is associative :

    a * b = ab+1

<u>Type – II (FOUR MARK QUESTIONS</u>)

88. Express $\begin{bmatrix} 6 & 1 & -5 \\ -2 & -5 & 4 \\ -3 & 3 & -1 \end{bmatrix}$ as the sum of a symmetric & a skew symmetric matrix.

89. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ using principle of M.I. show that :

$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix} \forall n \in N$

90. If f (x) = $x^2 - 5x + 7$, find f (A) if A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

91. Show that A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$. Hence find A^{-1}.

92. If a, b, c are in A.P. evaluate $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$

93. Using properties of determinants P.T. :

i. $\begin{vmatrix} x^2+1 & xy & zx \\ xy & y^2+1 & yz \\ zx & yz & z^2+1 \end{vmatrix} = 1 + x^2 + y^2 + z^2$

ii. $\begin{vmatrix} b+c & a+c & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

iii. $\begin{vmatrix} 1+a^2-b^2 & 2ab & 2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$.

iv. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

v. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+x \end{vmatrix} = xyz + xy + yz + zx$

vi. $\begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 1+3p+2q \\ 3 & 6+3p & 1+6p+3q \end{vmatrix} = 1$

vii. If x, y, z are different & $\begin{vmatrix} x & x^2 & 1+ax^3 \\ y & y^2 & 1+ay^3 \\ z & z^2 & 1+az^3 \end{vmatrix}$ = 0 prove that xyz = -1/a

94. If a, b, c are positive and are the p^{th}, q^{th}, r^{th} terms of a G.P., using properties of determinants prove

that : $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$

95. Using properties of determinants P.T.

i. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$ = 2abc $(a+b+c)^3$

ii. $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}$ = $(ab + bc + ca)^3$

96. Solve using properties of determinants

i. $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$

ii. $\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix} = 0$

iii. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$

97. Find a, b, c if A = $\frac{1}{3}\begin{bmatrix} a & 2 & 2 \\ 2 & 1 & b \\ 2 & c & 1 \end{bmatrix}$ & AA' = 1 where A' is transpose of matrix A.

98. If A = $\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find k s.t. $A^2 = kA - 2I$. Hence find A^{-1}.

99. Find the matrix $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$ $s.t. \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$

100. If A = $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, show that

i. A'A is symmetric

ii. A-A' is skew symmetric

Where A' denotes transpose of matrix A

101. Discuss continuity of f (x) = |x-1| + |x-2| at x = 1, 2.

102. Show that the function defined as :

F (x) = $\begin{cases} 3x-2 & 0<x\le 1 \\ 2x^2-x, & 1<x\le 2 \\ 5x-4 & x>2 \end{cases}$

Is continuous at x = 2 but not differentiable at x = 2.

103. Find a, b if :

i. f (x) = $\begin{cases} \dfrac{1-\sin^2 x}{3\cos^2 x}, & x<\pi/2 \\ a, & x=\pi/2 \\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, & x>\pi/2 \end{cases}$ is continuous at x = $\pi/2$

ii. f (x) = $\begin{cases} \dfrac{x-5}{|x-5|}+a, & x<5 \\ a+b, & x=5 \\ \dfrac{x-5}{|x-5|}+b, & x>5 \end{cases}$ is continuous at x = 5

104. Find the points of discontinuity (if any) for :

i. f (x) = $\begin{cases} |x|+3, & x\le -3 \\ -2x, & -3<x<3 \\ 6x+2, & x\ge 3 \end{cases}$

ii. f (x) = |x| - |x + 1|

iii. $f(x) = \begin{cases} 2\sqrt{x}, & 0 \le x \le 1 \\ 4-2x, & 1 < x < 5/2 \\ 2x-7, & 5/2 \le x \le 4 \end{cases}$

105. Find a, b if $f(x) = \begin{cases} x^2, & x \le c \\ ax+b, & x > c \end{cases}$ *is differentiable at* $x = c$.

106. Find p if $f(x) = \begin{cases} \dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & -1 \le x < 0 \\ \dfrac{2x+1}{x-2}, & 0 \le x \le 1 \end{cases}$ *is continuous* at x = 0

107. Find $\dfrac{dy}{dx}$ if :

i. $y = (\sin^{-1} x)^x + \sin^{-1} \sqrt{x}$

ii. $y = \tan^{-1} \left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}} \right]$ *for* $0 < x < \pi/2 \;\&\; \pi/2 < x < \pi$

iii. $y = \sin\text{-}1 \left(\dfrac{1-x^2}{1+x^2} \right)$, 0 < x < 1

iv. $y = \cos^{-1} \left(\dfrac{1-x^2}{1+x^2} \right)$ if 0 < x < 1 & - ∞ < x < 0

v. $y = \tan^{-1} \left(\dfrac{\cos x}{1+\sin x} \right)$

vi. $y = x^{x^x}$

vii. $x^y + y^x = 1$

viii. $x^y\, y^x = 1$

ix. $xy = e^{x-y}$

x. xy = tan (xy)

xi. $y = x^x - 2^{\sin x}$

xii. $xy + y^2 = \tan x + y$

xiii. $(x^2+y^2)^2 = xy$

xiv. $y = 10^{x^{10x}}$

xv. $y = \dfrac{\sin x}{1+\dfrac{\cos x}{1+\dfrac{\sin x}{1+\dfrac{\cos x}{1+.........\infty}}}}$

xvi. $y = \left(\dfrac{3+x}{1+x}\right)^{2+3x}$

xvii. $y = \sin^{-1}\left(\dfrac{5x+12\sqrt{1-x^2}}{13}\right)$

xviii. $y = \tan^{-1}\left[\dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$

108. If x $\sqrt{1+y}+y\sqrt{1+x}=0,$ *prove that* $\dfrac{dy}{dx}=\dfrac{-1}{(1+x)^2}$

109. If y = log $\left(\dfrac{x^2}{e^2}\right)$ *find* $\dfrac{d^2y}{dx^2}$

110. If y = sin (log x) P.T. $x^2y_2 + xy_1 + y = 0$.

111. If x = tan $\left(\dfrac{1}{a}\log y\right)$ P.T. $(1 + x^2)y_2 + (2x - a)y_1 = 0$

112. If y = $\dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$ P.T. $(1 - x^2)y_2 - 3xy_1 - y = 0$.

113. If y = $\left\{x+\sqrt{x^2+1}\right\}^m$, $P.T. (x^2+1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-m^2y=0$

114. If y = x^x P.T. $\dfrac{d^2y}{dx^2}-\dfrac{1}{y}\left(\dfrac{dy}{dx}\right)^2-\dfrac{y}{x}=0$

115. If x = a$\cos^3\theta$, y = a$\sin^3\theta$ find $\dfrac{d^2y}{dx^2}$ *at* $\theta=\pi/4$

116. If y = x^{n-1} logx, find the value of $x^2y_2 + (3 - 2n)xy_1$

117. If y = $ax^{n+1} + bx^{-n}$, Prove that : $x^2y_2 = n(n+1)y$

118. If x = sint & y = sinpt, prove that : $(1 - x^2)y_2 - xy_1 + p^2y = 0$

119. If $y = \log\left(\frac{x}{a+bx}\right)^x$, prove that : $x^3y_2 = (xy_1 - y)^2$

120. Differentiate

i. $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1}x$; $x \neq 0$

ii. sin-1 $\sqrt{1-x^2}\ w.r.t. \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$, $0 < x < 1$

121. Verify Rolles theorem for the following functions :

i. $f(x) = (x^2 - 1)(x-2)$ on $[-1, 2]$

ii. $f(x) = e^{-x} \sin x$ on $[0, \pi]$

iii. $f(x) = \sin^4 x + \cos^4 x$ on $[0, \pi/2]$

iv. $f(x) = x(x+2)e^{-x/2}$ on $[-3, 0]$

122. If the tangent to the curve $y = x^3 + ax + b$ at P (1, -6) is parallel to the line $y - x = 5$, find a & b.

123. At what points will the tangent to the curve $y = 2x^3 - 15x^2 + 36x - 21$ be parallel to the x-axis ? Also find the equations of the tangents to the curve at those points.

124. If the curves $y^2 = 4ax$ & $xy = c^2$ cut at right angles, prove that $c^4 = 32a^4$.

125. Show that the straight line $\frac{x}{a}+\frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point where the curve crosses the y - axis.

126. The pressure P & the volume V of a gas are connected by the relation $PV^{1/4} = a$ where is a is a constant. Find the % increase in the pressure corresponding to ½% decrease in volume.

127. Find the equation(s) of tangent (s) to the curve $y = x^3 + 2x + 6$ which are perpendicular to the line $x + 14y + 4 = 0$

128. Show that the curves $y = x^3 - 3x2 - 8x - 4$ & $y = 3x^2 + 7x + 4$ touch each other. Also find the equation of the common tangent.

129. A man of height 180 cm is moving away from a lamp post at 1.2 m/s. If the height of the lamp post is 4.5 m, find the rate at which his shadow is lengthening.

130. A man is moving away from a tower 85 m high at a speed of 4 m/s. Find the rate at which his angle of elevation of the top of the tower is changing when he is at a distance of 60m from the foot of the tower.

131. The volume of a spherical balloon is increasing at the rate of 4 cm^3/s. Find the rate of change of its surface area when its radius is 6 cm.

132. A ladder 13 m long leans against a wall. The foot of the ladder is pulled along the ground away from the wall at the rate of 1.5 m/s. How fast is the angle θ between the ladder & the ground changing when the foot of the ladder is 12 m away from the wall ?

132. Water is running into an inverted cone at the rate of π m^3/min. The height of the cone is 10 m & the radius of its base is 5 m. How fast is the water level rising when the water stands 7.5 m above the base ?

133. A particle moves along the curve $y = x^5 + 2$. Find the points on the curve at which the y co-ordinate changes five times as fast as the x co-ordinate.

134. Let I be the interval disjoint from (-1, 1). Prove that f (x) = x + 1/x is strictly increasing on I.

135. Solve the following differential equations –

i. $ydx + x \log\left(\frac{y}{x}\right) dy - 2xdy = 0$

ii. $y_1 - 3y \cot x = \sin 2x$; y = 2 when $x = \pi/2$

iii. $\frac{dy}{dx} + \frac{4x}{x^2+1} y = \frac{1}{(x^2+1)^3}$

iv. $(x + 2y^3)\, dy = ydx$

v. $\sqrt{1-y^2}\, dx = (\sin^{-1} y - x)$; y (0) = 0

vi. $\frac{dx}{dy} + x = 1 + e^{-y}$

vii. $(1 + x^2)\, dy + 2xy\, dx = \cot x\, dx$

viii. $(x + y)(dx - dy) = dx + dy$

ix. $xe^{y/x} - y + xy' = 0$, y (e) = 0

x. $\frac{dy}{dx} = (4x + y + 1)^2$

xi. $(x^3 + y^3)\, dy - x^2y\, dx = 0$

xii. $x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$

xiii. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1;(x\neq 0)$

xv. $ye^{x/y}\,dx = (xe^{x/y} + y)\,dy;\ y \neq 0$

136. Form the differential equation of the family of circles touching the y-axis at the origin.

137. Form the differential equation representing the family of curves given by $(x-a)^2 + 2y^2 = a^2$ where a is an arbitrary constant.

138. Verify that y = 3cos (log ×) + 4 sin (log ×) is a solution of the differential equation :

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$

139. Evaluate :

i. $\int\frac{3x-2}{(x+1)^2\,(x+3)}dx$

ii. $\int\frac{x+2}{2x^2+6x+5}dx$

iii. $\int_0^{\pi/4}\log\,(1+\tan x)\,dx$

iv. $\int_0^{\pi/2}\frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx$

v. $\int\frac{(x^2+1)}{(x+1)^2}dx$

vi. $\int_{-1}^{2} f(x)\,dx$ where $f(x) = |x+1| + |x| + |x-1|$

vii. $\int\frac{\sin 2x}{a\cos^2 x+b\sin^2 x}dx$

viii. $\int_0^{\pi/2}\frac{dx}{1+\sqrt{\cot x}}$

ix. $\int_0^{\pi/2}\left(\sqrt{\tan x}+\sqrt{\cot x}\right)dx$

x. $\int \frac{x^2+1}{x^4+x^2+1}dx$

xi. $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

xii. $\int_{-1}^{3/2} |x \sin \pi x|\, dx$

xiii. $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x \, dx$

xiv. $\int \frac{dx}{\sqrt{3-2x-x^2}}$

xv. $\int_0^{\pi/2} \frac{dx}{2\cos x+3\sin x}$

xvi. $\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2}+\frac{\pi}{4}\right) dx$

xvii. $\int \frac{1-x^2}{x(1-3x)}dx$

xviii. $\int \frac{x+3}{(x^2+1)(2x-1)}dx$

xix. $\int \frac{\sin x+\cos x}{\sqrt{\sin 2x}}dx$

xx. $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}dx$

xxi. $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$

xxii. $\int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$

xxiii. $\int \frac{dx}{\cos(x-a)\cos(x-b)}$

xxiv. $\int \frac{dx}{1-\tan x}$

xxv. $\int \cos 2x \cos 4x \cos 6x \, dx$

xxvi. $\int \frac{dx}{\sin x(3+2\cos x)}$

xxvii. $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}dx$

xxviii. $\int_0^1 \cot^{-1}(1-x+x^2)\,dx$

xxix. $\int_0^{\pi/2} \frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$

xxx. $\int_0^{\pi} \frac{x\tan x}{\sec x+\tan x}dx$

xxxi. $\int_0^{\pi} \frac{xdx}{1+\cos^2 x}$

xxxii. $\int_{-\pi/2}^{\pi/2} (\sin|x| - \cos|x|)\,dx$

xxxiii. $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

140. Prove that : $\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)dx, & If\ f(2a-x)=f(x) \\ 0, & if\ f(2a-x)=-f(x) \end{cases}$

141. Evaluate as limit of a sum :

$\int_1^3 (2x^2 + x)dx$

142. Prove that $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)\,dx = \frac{-\pi}{2}\log 2$

143. If $\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}$ such that f (2) = 0, Find f (x)

144. Evaluate :

i. $\int \frac{dx}{x^2(x^4+1)^{3/4}}$

ii. $\int \frac{dx}{x^{1/2}+x^{1/3}}$

iii. $\int \frac{dx}{x(x^5+1)}$

iv. $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4} dx$

v. $\int \frac{dx}{\sin^4 x+\cos^4 x}$

vi. $\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx$

145. Form the D.E. of the family of curves y = a sin (bx + c) if :

i. a, c are parameters

ii. a, b, c are parameters

146. Find a vector of magnitude 3 which is perpendicular to each of the vectors $4\hat{i}$ - $\hat{j}+3\hat{k}$ $\&-2\hat{i}+\hat{j}-2\hat{k}$

147. If $\vec{\alpha}=3\hat{i}-\hat{j}$ $\&\vec{\beta}=2\vec{i}+2\hat{j}-3\hat{k}$, express $\vec{\beta}$ as the sum of two vectors $\vec{\beta}\&\vec{\beta}_2$ *where* $\vec{\beta}_1 \parallel \vec{\alpha}$ $\&\vec{\beta}_2\perp\vec{\alpha}$

148. Three vectors $\vec{a},\vec{b},\vec{c},$ satisfy the condition of $\vec{a}+\vec{b}+\vec{c}=0$ Find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ *if* $|\vec{a}|=1,|\vec{b}|=4\,\&|\vec{c}|=2$

149. If $\vec{a}+\vec{b}+\vec{c}$ is a zero vector $\&|\vec{a}|=3,|\vec{b}|=5\,\&|\vec{c}|=7$ find angle between $\vec{a}\,\&\vec{b}$

150. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}\,\&\vec{b}=\hat{j}-\hat{k}$ find a vector $\vec{c}$ *s.t.* $\vec{a}\times\vec{c}=\vec{b}$ *and* $\vec{a}.\vec{c}=3$

151. Let $\vec{a}=\hat{i}-\hat{j},\vec{b}=3\hat{j}-\hat{k}$ $\&\vec{c}=7\hat{i}-\hat{k}$ Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}\,\&\vec{b}$ and $\vec{c}.\vec{d}=1$

152. If three vectors $\vec{a},\vec{b},\vec{c}$ are such that $\vec{a}+\vec{b}+\vec{c}$ *is a zero vector prove that* $\vec{a}\times\vec{b}=\vec{b}\times\vec{c}=\vec{c}\times\vec{a}$

153. For any two vectors $\vec{a}\,\&\vec{b}$ show that :

$$(1+|\vec{a}|^2)(1+|\vec{b}|^2)=(1-\vec{a}.\vec{b})^2+|\vec{a}+\vec{b}+(\vec{a}\times\vec{b})|^2$$

154. If $\vec{a},\vec{b},\vec{c}$ *are vectors s.t.* $\vec{a}.\vec{b}=\vec{a}.\vec{c}$ *and* $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ *and* $\vec{a}\neq\vec{0}$ then prove that $\vec{b}=\vec{c}$

155. If $\vec{a},\vec{b},\vec{c}$ *represent the vectors* $B\vec{C},C\vec{A},A\vec{B}$ *of a* $\Lambda\,ABC$

Prove that : $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

156. If $\vec{a},\vec{b},\vec{c}$ are position vectors of the vertices of a triangle, prove that vector area of the triangle is given by : $\frac{1}{2}\left(\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right)$

157. Show that $(\vec{a}-\overline{b})\times(\vec{a}\times\vec{b})=2(\vec{a}\times\vec{b})$ and interpret the result geometrically. Find the area of a ǁ gm whose diagonals are represented by the vectors $2\hat{i}-3\hat{j}+4\hat{k}\,\&-3\hat{i}+4\hat{j}-\hat{k}$

158. Find x, y if $3\hat{i}+x\hat{j}-\hat{k}$ & $2\hat{i}+\hat{j}+y\hat{k}$ are perpendicular vectors of equal magnitudes.

159. Let $\vec{a}=4\vec{i}+5\hat{j}-\hat{k},\vec{b}=\hat{i}-4\hat{j}+5\hat{k}$ & $\vec{c}=3\hat{i}+\hat{j}-\hat{k}$ *Find a vector* $\vec{d}$ *s.t.* $\vec{d}\perp\vec{a},\vec{d}\perp\vec{b}\,\&\,\vec{d}.\vec{c}=21$

160. If $\vec{a},\vec{b},\vec{c}$ are three mutually perpendicular vectors of equal magnitudes, show that $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to the vectors $\vec{a},\vec{b}$ & $\vec{c}$

161. Find the co-ordinates of the foot of perpendicular drawn from the point (1, 2, 1) to the line joining the points (1, 4, 6) & (5, 4, 4). Also find the length of perpendicular.

162. Find the point on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point (1, 2, 3)

163. Find the equation of the plane parallel to the line $\frac{x-2}{1}=\frac{y-1}{3}=\frac{z-3}{2}$ which contains the point (5, 2, -1) and passes through the origin.

164. Find the distance of the point (1, 2, -3) from the plane x – y + z = 5 measured parallel to the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-6}$

165. Find the shortest distance between the lines;

$$\vec{r}=(6\vec{i}+2\vec{j}+2\hat{k})+\lambda(i-2\hat{j}+2\hat{k})$$

$$\vec{r}=4\vec{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$$

166. Find the vector equation of the plane passing through the intersection of the planes 2x + 3y – z + 1 = 0. x + y – 2z + 3 = 0 and perpendicular to the plane 3x – y – 2z – 4 = 0. Also, find the inclination of this plane with the xy- plane.

167. Find the distance of the point (-1, -5, 10) from the point of intersection of the line :

$$\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+12\hat{k})\,\&\,\textit{the plane}\;\vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$$

168. Find the Cartesian & vector equation of the plane passing through the intersection of the planes $\vec{r}.(2\hat{i}+6\hat{j})+12=0 \& \vec{r}.(3\hat{i}-\hat{j}+4\hat{k})=0$ which are at unit distance from the origin.

169. Find the co-ordinates of the foot of perpendicular drawn from origin to the plane 2x – 3 y + 4z – 6 =0

170. Find the equation of the plane through the points (2, -3, 1) & (5, 2, -1) and perpendicular to the plane x – 4y + 5z + 2 = 0

171. Prove that the angle between any two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$

172. Show that the lines

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta}$$

and $\frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$ are coplanar

Also find the equation of the plane containing these lines.

173. Find the vector equation of the line passing through the point (1, 2, 3) & parallel to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5, \vec{r}.(3\vec{i}+\hat{j}+\hat{k})=6$

174. Find the vector equation of the plane containing the lines :

$$\vec{r}=3\hat{i}-\hat{j}+4\hat{k}+t(2\hat{i}-\hat{k})$$

& $\vec{r}=\hat{i}+\hat{j}-2\hat{k}+s(\hat{j}+2\hat{k})$

175. Find the points on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance of 5 units from the point P (1, 3, 3)

176. Find the distance of the point (6, 5, 9) from the plane determined by the points A (3, -1, 2), B (5, 2, 4) & C (-1, -1, 6).

177. Let N be the set of natural numbers & R be a relation on N × N defined by (a, b) R (c, d) $\Leftrightarrow$ ad = bc for all (a, b), (c, d) EN × N. Show that R is an equivalence relation on N × N.

178. Let N be the set of all natural numbers & R be the relation on N × N defined by (a, b) R (c, d) iff ad (b + c) = bc (a + d) Examine whether R is an equivalence relation on N × N.

179. Show that the relation R on the set A = $\{xEZ:0\leq x\leq 12\}$ given by R = {(a, b) : la-bl is a multiple of 3} is an equivalence relation. Find the set of all elements related to 1.

180. Show that the relation R on the set $A = \{1, 2, 3, 4, 5\}$ given by : R {a, b) |a – b| is even} is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other but no element of {1, 3, 5} is related to any element of {2, 4}

181. Show that the relation R in the set of real numbers defined as :

 $R = \{(a, b) : a \leq b^3\}$ is neither reflexive, nor symmetric nor transitive.

182. Find if the relation R defined in the set A = {1, 2, 3, ………. 13, 14} defined as : R = {(x, y) : 4x – y = 0} is reflexive, symmetric & transitive.

183. Let A = {1, 2, 3}. Find the no. of relations containing (1, 2) & (2, 3) which are reflexive & transitive but not symmetric.

184. Let A = {1, 2, 3}. Find the no. of equivalence relations in A containing (1, 2) & (2, 1)

185. Let R be the relation defined on the set A = {1, 2, 3,4, 5, 6, 7} by R = {(a, b) : both a & b are either even or odd}. Show that R is an equivalence relation. Further show that all elements of the subset {1, 3, 5, 7} are related to each other & all elements of the subset {2, 4, 6} are also related to each other but no element of the subset {1, 3, 5, 7} is related to any element of the subset {2, 4, 6}.

186. Let A = {a, b, c} & R be the relation defined on A as follows :

 R = {(a, a), (a, b), (a, c), (b, b), (b, c), (c, a), (c, b), (c, c)}

 Find whether R is reflexive, symmetric & transitive.

187. Let L be the set of all lines in XY plane and R be the relation in L defined as :

 $R = \{L_1, L_2) : : L_1$, is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line y = 3x + 5.

188. Show that the operation* on Q – {1} defined by :

 $a * b = a+b - ab \ \forall a,b \in Q - \{1\}$ is :

 i. commutative

 ii. associative

 Also find the identity element and inverse for the operation* on Q – {1}.

189. Let A = N × N & * be a binary operation on A defined as : (a, b) * (c, d) = (a + c, b+d).

 Show that * is commutative & associative. Find the identity element of * on A (if any).

190. Consider the binary operation * on the set {1, 2, 3, 4, 5} defined as : a * b = h.c.f. of a & b. Write the operation table & find if * is

 i. commutative

ii. associative

Also find if identity element exists for * on the given set.

191. Let A = R – {2} & B = R – {1}. If f : A→B is define by f (x) = $\frac{x-1}{x-2}$, show that f is a bijection. Also find f^1.

192. Find if f : Q – {3} → Q defined as f (x) = $\frac{2x+3}{x-3}$ is :

i. One-one

ii. onto

193. Show that f : [-1, 1] → R given by f (x) = $\frac{x}{x+2}$ is one-one onto. Also find inverse of the function f : [-1, 1) → Range (f).

194. Consider F : R + → [4, ∞) given by : f (x) = x^2 + 4 where R + is the set of all non-negative and numbers. Show that f is invertible & find f^1.

195. Show that f : R – {-1} → R – {1} given by f (x) = $\frac{x}{x+1}$ is invertible. Also find f^1.

196 Let f : R - $\left\{-\frac{4}{3}\right\}$ → R be a function defined as :

f(x) = $\frac{4x}{3x+4}$. Is f invertible ? Find f^{-1} : Range f → R - $\left\{-\frac{4}{3}\right\}$. What is the range of f ?

197. Let R be a relation defined on the set R of real numbers as : R = {(a, b) ∈ R × R; a < b}. Find if the relation R is reflexive, symmetric, transitive.

198. Let * be a binary operation on the set of integers given as :

a*b = a + b + 1 ∀ a, b, ∈ Z

Is * (i) commutative (ii) associative

Find the identity element for * on I. Also find × if 3 * (x * 4) = 11.

199. Prove that : $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$

200. i. Simplify : $\cot^{-1}\left(\sqrt{1+x^2}-x\right)$

ii. Solve : $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\text{cosec}\, x)$

201. Evaluate : $\cos\left(\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$

202. Prove that : $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

203. An operation * defined on the set of positive rational numbers Q+ is given by :

a*b = $\frac{ab}{2}\forall a,b\in Q^{+}$

Prove that * is a binary operation on Q^+. Find the identity element for * in Q^+. Also the inverse of a $\in$ Q^+.

204. Prove that :

$$\frac{9\pi}{8}-\frac{9}{4}\sin^{-1}\frac{1}{3}=\frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

205. Simplify :

i. $\tan^{-1}\left\{x+\sqrt{1+x^2}\right\}$

ii. $\sin^{-1}\left\{x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right\}$

206. Prove that : $\text{Sin}^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

207. Solve for x :

ii. $\tan(\cos^{-1}x) = \sin\cot^{-1}\left(\frac{1}{2}\right)$

iv. $\tan^{-1}2x + \tan^{-1}3x = \pi/4$

208. Evaluate :

$$2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)$$

209. Prove that :

$$\text{Cot}^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2}$$

210. Two cards are drawn successively from a well shuffled pack of 52 cards. Find the probability distribution of the no. of aces. Also find mean & standard deviation.

212. In a school there are 1000 students out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl ?

213. A bag X contains 2 white & 3 red balls and a bag Y contains 4 white & 5 red balls. One ball is drawn at random from one of the bags & is found to be red. Find the probability that it was drawn from bag Y.

214. 12 cards numbered 1 to 12 are placed in a box, mixed up thoroughly and then a card is drawn at random from the box. If it is known that the number on the drawn card is more than 3, find the probability that it is an even number.

215. In a multiple choice examination with three possible answers (out of which only one is correct) for each of the five questions, what is the probability that a candidate would get four or more correct answer just by guessing ?

216. A bag contains 11 tickets numbered 1 to 11. Two tickets are drawn without replacement. What is the probability that the second ticket has an even number given that the first has an odd number ?

217. In a hurdle race a player has to cross 10 hurdles. The probability that he will clear each hurdle is 2/3. What is the probability that he will knock down fewer than 3 hurdles.

218. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn & are found to be spades. Find the probability of the lost card being a Spade.

219. A speaks the truth 8 times out of 10 times. A die is tossed. He reports getting a 5. What is the probability that it was actually a 5 ?

220. The probability of a shooter hitting a target is ¾. How many minimum no. of times must he/she fire so that the probability of hitting the target at least once is more than 0.99 ?

221. Bag 1 contains 2 red & 3 black balls. Bag II contains 3 red & 5 black balls. One ball is transferred from bag I to bag II & then a ball is drawn from bag II. The ball so drawn is found to be black in colour. Find the probability that the transferred ball is red.

222. A man takes a step forward with probability 0.2 & backward with probability 0.8. Find the probability that at the end of eleven steps, he is one step away from the starting point.

223. A can hit a target 3 times out of 5 times, B can hit a target 2 times out of 5 times & C an hit 3 times out of 4 times. Find the probability that two out of A, B and C will hit the target.

224. A student takes his examination in 4 subjects A, B, C & D. To qualify he must pass in A and at least 2 other subjects. His chances of passing in A, B, C & D are 4/5, ¾, 5/6, 2/3 respectively. Find the chances of his qualifying.

225. There are 2 bags A & B containing 3 black & 4 white; 4 black & 3 white balls respectively. A die is thrown. If 3 or 5 turns up, a ball is drawn from bag A. Otherwise a ball is drawn from bag B. Find the probability of getting a black ball.

226. A is known to tell the truth in 5 cases out of 6 and he states that a white ball was drawn from a bag containing 9 red & 1 white ball. Find the probability that a white ball was drawn.

227. In a school 8% of the girls & 2% of the boys have an intelligent quotient of more than 120. In the school 60% of the students are girls. A student with intelligent quotient more than 120 is selected. Find the probability that the student selected is a girl.

228. A family has 2 children. Find the probability that both are boys, if it is known that :

a. at least one of the children is a boy.

b. the elder child is a boy.

Type – III (SIX MARK QUESTIONS)

229. Using matrix method solve the following system of equations :

$5x + y - z = 7$

$4x - 2y - 3z = 5$

$7x + 2y + 2z = 7$

230. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, find A^{-1}. Hence solve the following system of equations:

$x + y + 2z = 0$

$x + 2y - z = 9$

$x - 3y + 3z + 14 = 0$

231. Use the product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations :

$x - y + 2z = 1$

$2y - 3z = 1$

$3x - 2y + 4z = 2$

232. Using elementary row transformations find the inverse of :

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

233. Using elementary column transformations, find the inverse of : $\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

234. Find the co-ordinates of the points on the curve $y = x^2 + 3x + 4$, the tangents at which pass through the origin. Also find the equations of the tangents.

235. The curve $y = ax^3 + bx^2 + cx + 5$ touches the x-axis at P (-2, 0) & cuts the y-axis at the point Q where its gradient is 3. Find the equation of the curve completely.

236. If the curves $4x = y^2$ & $4xy = k$ cut at right angles show that $k^2 = 512$.

237. Evaluate :

i. $\int \frac{\sqrt{\cos 2x}}{\sin x} dx$

ii. $\int \frac{dx}{\sin x + \sec x}$

iii. $\int_0^a \sin^{-1} \sqrt{\frac{x}{a+x}}\, dx$

iv. $\int_0^\pi \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

v. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx.$

238. Prove that : $\int_0^{\pi/2} \log \sin x\, d\, x = -\frac{\pi}{2} \log 2$

239. Evaluate as limit of a sum :

$\int_0^3 (2x^2 + 3x + 5) dx$ and $\int_1^3 (x^2 - x + 2)\, dx$

240. A window is in the form of a rectangle surrounded by a semicircle. If the perimeter of the window is P cm, show that the window will allow the maximum possible light only when the radius of the semicircle is $\frac{P}{\pi + 4} cm.$

241. A box of constant volume C is to be twice as long as it is wide. The material on the top & four sides costs three times as much per square m as that in the bottom. What are the most economical dimensions ?

242. A square tank of capacity $250m^3$ has to be dug out. The cost of land is Rs. 50 per sq. m. The cost of digging increases with the depth & for the whole tank is 400 $(\text{depth})^2$ rupees. Find the dimensions of the tank for the least total cost.

243. The perimeter of a rectangle is 100 m. Find the length of its sides when the area is maximum.

244. A box is constructed from a rectangular metal sheet 21 cm by 16 cm by cutting squares of sides x cm from the corners of the sheet & then turning up the projected portions. For what values of x, will the volume of the box be maximum.

245. The section of a window consists of a rectangle surmounted by an equilateral triangle. If the perimeter be given as 16 m, find the dimensions of the window in order that the maximum light may be admitted.

246. If the sum of lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is $\pi/3$.

247. Find the maximum area of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis.

248. A point on the hypotenuse of a triangle is at distance a & b from the sides. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

249. Show that the right circular cone of least curved surface area & given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

250. Show that the semi-vertical angle of a right circular cone of given surface area & maximum volume is Sin^{-1} (1/3).

251. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h & semi-vertical angle 30^0 is $\frac{4}{81}\pi h^3$.

252. An open box with a square base is to be made out of a given quantity of sheet of area a^2. Show that the maximum volume of the box is $\frac{a^3}{6\sqrt{3}}$.

253. The total area of a page is 300 cm^2. The combined width of the margin at the top & bottom is 6 cm & the side is 4 cm. What must be the dimensions of the page in order that the area of the printed matter is maximum ?

254. Using integration find the area of the region :

$\{(x, y) : y^2 \leq 3x, 3x^2 + 3y^2 \leq 16\}$

255. Find the area bounded by the curve $y = x\,|x|$, x-axis and the lines x = -1,, x = 1 using integration.

256. Sketch the region bounded by the curves $y = \sqrt{5-x^2}$ & $y = |x - 1|$. Find its area using method of integration.

257. Find the area of the region in the 1st quadrant enclosed by the x-axis, the line y = x & the circle $x^2 + y^2 = 32$.

258. Using integration find the area bounded by the curves y = sin x, y = cos x & the x – axis s.t. $0 \le x \le \pi/2$

259. Using integration find the area enclosed between the circles $x^2 + y^2 = 1$ & $(x - 1)^2 + y^2 = 1$.

260. Find integration find the area enclosed by the given curves :

i. $y^2 = x + 1$ & $y^2 = -x + 1$

ii. $4y = 3x^2$ & $2y = 3x + 12$

iii. $x^2 + y^2 = 25$ & $x + y = 5$

iv. $y = 6x - x^2$ & $y = x^2 - 2x$

v. $y = 2x - x^2$ & $y = -x$

261. Using integration find the area bounded by the lines :

i. x + 2y = 2, y – x = 1 & 2x + y = 7

ii. 2x + y = 4, 3x – 2y = 6 & x – 3y + 5 = 0

262. Using integration find the area of the region :

i. $\{x, y) : x^2 + y^2 \le 1 \le x + y/2\}$

ii. $\{x, y) : 0 \le y \le x^2 + 3, 0 \le y \le 2x + 3, 0 \le x \le 3\}$

iii. $\{(x, y) : y^2 \ge 6x, x^2 + y^2 \le 16\}$

iv. $\{(x, y) : 9x^2 + y^2 \le 36, 3x + y \ge 6\}$

263. Using integration find the area of ΛABC with vertices A (-1, 0), B (1, 3) & C (3, 2)

264. Using integration find the area of the region bounded by the curve $|x| + |y| = 1$

265. Using integration find the area bounded by the curves :

i. $x^2 = 4y$ & $x = 4y - 2$

ii. $x^2 = 4y$, $x = 4y - 2$ & the x-axis

266. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

267. Find the area enclosed between the circle $x^2 + y^2 = 16$, the parabola $x^2 = 6y$ & the y-axis.

268. It is given that the rate at which some bacteria multiply is proportional to the instantaneous number present. If the original no. of bacteria doubles in two hours, in how many hours will it be five times ?

269. Find the image of the point (1, 2, 3) in the plane x + 2y + 4z = 38.

270. Find the vector equation of the line of shortest distance between the lines :

$$\frac{x-6}{1}=\frac{y-2}{-2}=\frac{z-2}{2} \;\&\; \frac{x+4}{2}=\frac{y}{-4}=\frac{z+1}{4}$$

Also find the S.D. between the lines.

271. Show that the points (0, -1, -1), (4, 5, 1), (3, 9, 4) & (-4, 4, 4) are coplanar. Write the vector equation & Cartesian equation of the common plane.

272. Mona wants to invest at the most Rs. 12,000 in savings certificate and National Savings Bonds. She has to invest at least Rs. 2000 in savings certificates and at least Rs. 4000 in National Saving Bonds. If the rate of interest on savings certificate is 8% p.a. and the rate of interest on national savings bonds 10% p.a., how much money should she invest to earn maximum yearly income ?

273. A brick manufacturer has two depots A & B with stocks of 50,000 & 25,000 bricks respectively. He receives orders from three builders P, Q & R for 30,000, 25,000 & 20,000 bricks respectively. The cost (in Rs.) of transporting 1000 bricks to the builders from the depots is given below :

To →	P	Q	R
From ↓			
A	60	30	25
B	40	20	30

How should the manufacturer fulfil the orders so as to keep the cost of transportation minimum ?

274. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each executive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passengers prefer to travel by economy class as by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit ?

275. A man owns a field area of 1000 sq. m. He wants to plant fruit trees in it. He has a sum of Rs. 1400 to purchase young trees. He has the choice of two types of trees.

Type A requires 10 sq. m. of ground per tree & costs Rs. 20 per tree and type B requires 20 sq. m. of ground per tree & costs Rs. 25 per tree. When fully grown, type A produce an average of 20 kg of fruit which can be sold at a profit of Rs. 2 per kg. and type B produces an average of 40 kg. fruits which can be sold at a profit of Rs. 1.50 per kg. How many trees of each type should be planted to achieve a maximum profit when the trees are fully grown ? What is that profit ?

276. A dietician has to develop a special diet using two foods P & Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol & 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin, A. The diet requires at least 240 units of Calcium, at least 460 units of iron and at the most 300 units of cholesterol. How many packets of each food should be used to minimize the amount of vitamin A in the diet ? What is the minimum amount of vitamin A ?

277. A toy company manufacturers two types of dolls, A & B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs. 12 & Rs. 16 per doll respectively on dolls A & B, how many of each type should be produced weekly in order to maximize the profit?

278. A small firm manufactures items A & B. The total no. of items A & B that it can manufacture in a day is at the most 24. Item A requires one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A is Rs. 300 and on one unit of item B is Rs. 160, how many of each type should be produced to maximize the profit ? Solve the problem graphically.

279. A home decorator manufacturers two types of lamps – A & B. Lamp A requires 2 hours of the cutter's time and 1 hour of the finisher. Lamp B requires 1 hour of the cutter's and 2 hours of the finisher's time. The cutter has 104 hours and the finisher has 76 hours of time available each month. Profit on each lamp of type A & B is Rs. 6, Rs. 11 respectively. Assuming that he can sell all that he produces, how many lamps of each type should be manufactured to obtain maximum profit ? Solve graphically.

280. Given three identical boxes I, II & III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins & in box III, there is one gold & one silver coin. A person chooses a box at random & takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also a gold coin ?

281. Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail.'

282. Four defective articles are mixed with ten good ones. A sample of 3 is drawn at random from the lot. Find the probability distribution of no. of defective articles drawn. Hence find mean & variance of the probability distribution.

283. By examining the chest X-ray, the probability that T.B. is detected when a person is actually suffering is 0.99. The probability of incorrect diagnosis is 0.001. In a certain city, 1 in 1000 persons suffers from T.B. A person is selected at random & is diagnosed to have T.B. What is the chance that he actually has T.B. ?

284. A class has 15 students whose ages (in years) are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 & 20. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X ? Find mean, variance & standard deviation of X.

285. A factory has 3 machines X, Y, Z producing 1000, 2000 & 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% & Z produces 2% defective bolts. At the end of a day, a bolt is drawn at random & is found defective. What is the probability that this defective bolt has been produced by machine X ?

286. A & B are playing a game. A throws a die & B tosses a coin turn by turn. A wins the game if he gets a number more than 4 on the die & B wins if he gets a head. Find their respective chances of winning if A starts the game.

287. Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation & yoga course reduces the risk of heart attack by 30% & prescription of certain drug reduces its chance by 25%. At a time a patient can choose any one of the two options, with equal probabilities. It is given that after going through one of the two options, the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation & yoga.

288. If a fair coin is tossed 10 times, find the probability of getting :

   i. exactly six heads

   ii. at least six heads

   iii. at most six heads

289. The fuel charges for running a train are proportional to the square of the speed generated in miles per hour and costs Rs. 48 per hour at 16 miles per hour. What is the most economical speed if the fixed charges are Rs. 500 per hour ?

290. A running track of 440 feet is to be laid out enclosing a football field, the shape of which is a rectangle with a semi-circle at each end. If the area of the rectangular portion is to be maximum, find the length of its sides.

291. When travelling x km/hour a truck burns diesel at the rate of [(900/x) + x] / 300 litres per km. If the diesel oil costs 40 paise per litre and the driver is paid Rs. 1.50 per hour, find the steady speed that will minimize the total cost of the trip of 500 km.

292. A given quantity of metal is to be cast into a half cylinder with a rectangular base and semi-circular ends. Show that in order that the total surface area may be minimum , the ratio of the length of the cylinder to the diameter of its semi-circular ends is $\pi : (\pi+2)$