Solutions – Pre-Board Exam – II – 2019-20
Subject – Mathematics
Class – XII

1 (a), 2 (c), 3 (c), 4 (b), 5 (c), 6 (b), 7 (b), 8 (c), 9 (a)
10 (b), 11) $x = -4$ 12) $\frac{1}{4}$ 13) $|r|$ units 14) 6 or $-2/3$ OR $x=1$
15) $z = 6$

16) Let $I = \int \frac{dx}{x \cdot x^2(1+x^{-2})} = \int \frac{x^{-3}\,dx}{1+x^{-2}}$

Let $x^{-2} = t \Rightarrow -2x^{-3}dx = dt$

$\therefore I = -\frac{1}{2}\int \frac{dt}{1+t} = -\frac{1}{2}\log|1+t| + C = -\frac{1}{2}\log\left|1+\frac{1}{x^2}\right| + C$

17) $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\cancel{5/2}\times 2} = \left(\frac{d^3y}{dx^3}\right)^2$

$\boxed{\text{Order} \to 3\ ,\ \deg \to 2.}$ Ans.

OR

D.E is: $\frac{dy}{dx} + \frac{1}{x\log x}y = \frac{2}{x}$ which is a LDE of the type $\frac{dy}{dx} + Py = Q$

I.F $= e^{\int P dx} = e^{\int \frac{1}{x\log x}dx}$ → Let $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$\therefore$ I.F $= e^{\int \frac{dt}{t}} = e^{\log t} = t = \boxed{\log x}$ Ans.

18)

$y = x+1$ —① (given line)
$\Rightarrow m_L = 1$

& $y^2 = 4x$ is the eqn of the given curve.
diffn b/s w.r.t. x we get:
$\cancel{2}y\frac{dy}{dx} = \cancel{4}\,2$
or $\frac{dy}{dx} = \frac{2}{y}$

Let the pt of contact be (x_1, y_1)
$\therefore m_T = \frac{dy}{dx}\Big]_{(x_1,y_1)} = \frac{2}{y_1} = 1$
$\Rightarrow y_1 = 2$
$\therefore x_1 = \frac{y_1^2}{4} = 1$

$\therefore$ Reqd Pt is $(1, 2)$ Ans.

18-OR
$\llcorner$, $a > 0$ for $x \in R$ Ans.

19) $I = \int \frac{xe^x}{(1+x)^2}dx = \int e^x\left(\frac{x+1-1}{(x+1)^2}\right)dx = \int e^x\left[\frac{1}{(x+1)} - \frac{1}{(x+1)^2}\right]dx$

Let $f(x) = \frac{1}{x+1}$
$\Rightarrow f'(x) = \frac{-1}{(x+1)^2}$ $\therefore I = \frac{e^x}{x+1} + C$ Ans

$\left(\because \int e^x(f(x)+f'(x))dx = e^x f(x) + C\right)$

20) $\frac{\pi}{2} - 2\cos^{-1}x = \pi/6 \Rightarrow \frac{\pi}{2} - \frac{\pi}{6} = 2\cos^{-1}x$

$\Rightarrow \frac{\pi}{3} = 2\cos^{-1}x$ or $\pi/6 = \cos^{-1}x$

or $x = \cos\pi/6 = \frac{\sqrt{3}}{2}$ Ans

21) LHS= $\cos^{-1}x + \cos^{-1}\left\{\frac{x}{2} + \frac{\sqrt{3}}{2}\sqrt{1-x^2}\right\}$ Sec-B

$= \cos^{-1}x + \cos^{-1}\left\{x \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \sqrt{1-x^2}\right\}$

Let $x = \cos\theta$ & $\frac{1}{2} = \cos\phi$

$\therefore$ LHS $= \theta + \cos^{-1}\{\cos\theta\cos\phi + \sin\theta\sin\phi\}$.

$= \theta + \cos^{-1}\{\cos(\phi - \theta)\}$

$= \theta + \phi - \theta$

$= \phi = \cos^{-1}(1/2)\ \pi/3 =$ RHS.

OR.

Here $R = \{(1,22), (2,20), (3,18), (4,16), (5,14), (6,12), (7,10), (8,8), (9,6), (10,4), (11,2)\}$

$y = 24 - 2x$; $x, y \in N$.

$\Rightarrow$ R is not symmetric as $(1,22) \in R$ but $(22,1) \notin R$.

Also R is not transitive as $(9,6) \in R$ & $(6,12) \in R$ but $(9,12) \notin R$

22) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{e^t\cos t - e^t\sin t}{e^t\cos t + e^t\sin t} = \frac{(\cos t - \sin t)/\cos t}{(\cos t + \sin t)/\cos t}$

or $\frac{dy}{dx} = \frac{1-\tan t}{1+\tan t} = \tan(\pi/4 - t)$

$\therefore \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx} = \sec^2\left(\frac{\pi}{4} - t\right)(-1) \times \frac{1}{e^t\cos t + e^t\sin t}$

$\therefore \left.\frac{d^2y}{dx^2}\right]_{x=\pi} = \frac{(-1)\sec^2(-3\pi/4)}{e^{\pi}(-1) + e^{\pi}(0)} = \frac{(+1)\sec^2(3\pi/4)}{+e^{\pi}}$

$= e^{-\pi}(-\sqrt{2})^2 = 2e^{-\pi} = \frac{2}{e^{\pi}}$ Ans.

23) $\frac{dA}{dt} = \frac{d(2r)}{dt}$

$\Rightarrow \frac{d}{dt}(\pi r^2) = 2\frac{dr}{dt}$ ($\because$ Area of circle $= \pi r^2$)

$\Rightarrow \pi \cdot \cancel{2} r \frac{\cancel{dr}}{\cancel{dt}} = \cancel{2}\frac{\cancel{dr}}{\cancel{dt}}$

$\Rightarrow r = \frac{1}{\pi}$ units.

24) $\vec{a} = \lambda \vec{b} = \lambda(6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k})$

$|\vec{a}| = 50 \Rightarrow \sqrt{36\lambda^2 + 64\lambda^2 + \frac{225}{4}\lambda^2}$

$\Rightarrow |\vec{a}| = \sqrt{\frac{625\lambda^2}{4}} = \pm\frac{25}{2}\lambda$

or $\pm \cancel{50}^2 = \frac{\cancel{25}}{2}\lambda \Rightarrow \lambda = \pm 4$

$\therefore \vec{a} = \pm 4(6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}) = \pm(24\hat{i} - 32\hat{j} - 30\hat{k})$

Also given that $\vec{a}$ makes an acute angle with the +ve dirn of z axis $\Rightarrow \vec{a} = -24\hat{i} + 32\hat{j} + 30\hat{k}$

OR

Reqd vector $= \pm \frac{6(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

Here $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 4\hat{i} - \hat{j}(2-1) + \hat{k}(-4-2)$
$= 4\hat{i} - \hat{j} - 6\hat{k}$

$|\vec{a}\times\vec{b}| = \sqrt{16+1+36} = \sqrt{53}$

$\therefore$ Reqd vector is $\pm\frac{6(4\hat{i} - \hat{j} - 6\hat{k})}{\sqrt{53}}$

25) C. eqn of plane is: $A(x-2) + B(y-3) + C(z-1) = 0$

Plane $\perp$ given line

$\Rightarrow$ normal to the plane $\parallel$ given line

$\Rightarrow A = 1, B = -1$ & $C = 2$

$\therefore$ C eqn of plane is: $x - 2 - y + 3 + 2z - 2 = 0$ i.e $x - y + 2z - 1 = 0$
& V. eqn is $\vec{r}\cdot(\hat{i} - \hat{j} + 2\hat{k}) = 1$

OR $E(X) = \sum X \cdot P(X)$
$= (0)\left(\frac{12}{13}\times\frac{12}{13}\right) + (1)\left(\frac{12}{13}\times\frac{1}{13}\right) + (2)\left(\frac{1}{13}\times\frac{1}{13}\right) = 2/13$

26) Here $X = 0, 1, 2$ where $X \to$ no. of aces drawn.

$P(X) = \frac{\cancel{4}\,1}{\cancel{52}\,13}$

$P(\bar{X}) = \frac{\cancel{48}\,12}{\cancel{52}\,13}$

X	$P(X)$		$X\,P(X)$
0	$P(\bar{X}\bar{X}) = \frac{12}{13}\times\frac{12}{13}$	$= \frac{144}{169}$	0
1	$P(X\bar{X})\times 2 \quad \frac{1}{13}\times\frac{12}{13}\times 2$	$= \frac{24}{169}$	$24/169$
2	$P(XX) = \frac{1}{13}\times\frac{1}{13}$	$= \frac{1}{169}$	$2/169$
		$\sum P(X) = 1$	$\sum X\,P(X) = \frac{\cancel{26}\,2}{\cancel{169}\,13} = E(X)$ Ans.

Section - C

27) For One-One Let $f(x_1) = f(x_2)$; $x_1, x_2 \in$ dom f.

$\Rightarrow \frac{2x_1+5}{3x_1-1} = \frac{2x_2+5}{3x_2-1}$

$\Rightarrow \cancel{6x_1x_2} - 2x_1 + 15x_2 - \cancel{5} = \cancel{6x_1x_2} - 2x_2 + 15x_1 - \cancel{5}$

$\Rightarrow 17x_2 = 17x_1$

or $x_1 = x_2$

$\Rightarrow f$ is one-one

For Onto Let $y \in R - \left\{\frac{2}{3}\right\}$

$y = f(x)$

$\Rightarrow y = \frac{2x+5}{3x-1}$

$\Rightarrow 3xy - y = 2x + 5$

$\Rightarrow 3xy - 2x = y + 5$

or $x(3y-2) = y+5$

$\Rightarrow x = \frac{y+5}{3y-2} \in R - \left\{\frac{1}{3}\right\}$.

If $x = 1/3$
$\frac{1}{3}(3y-2) = y+5$
$\Rightarrow \cancel{y} - \frac{2}{3} = \cancel{y} + 5$
$\Rightarrow -2/3 = 5$ which is NOT possible

$\forall\, y \in$ codomain f, $\exists$ a pre-image in domain f

$\Rightarrow f$ is onto

As f is one-one onto, it is invertible. f^{-1} exists & is defined as: $f^{-1} : R - \left\{\frac{2}{3}\right\} \to R - \left\{\frac{1}{3}\right\}$

s.t. $\boxed{f^{-1}(y) = \frac{y+5}{3y-2}}$

27 OR

Reflexivity: $(a,b)\,R\,(a,b) \quad \forall \quad a,b \in A$

$\because \quad a+b = b+a \quad \forall\, a,b \in A$

$\Rightarrow$ R is reflexive.

Symmetry: Let $(a,b)\,R\,(c,d)$

$\Rightarrow \quad a+d = b+c \quad ; a,b,c,d \in A$

$\Rightarrow \quad d+a = c+b$

$\Rightarrow \quad c+b = d+a$

$\Rightarrow \quad (c,d)\,R\,(a,b)$

$\therefore$ R is symmetric

Transitive: Let $(a,b)\,R\,(c,d)$ & $(c,d)\,R\,(e,f)$

$\Rightarrow \quad a+d = b+c$ & $c+f = d+e$

$\Rightarrow \quad a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$

or $\quad a+f = b+e$

$\Rightarrow \quad (a,b)\,R\,(e,f)$

$\therefore$ R is transitive.

Since R is reflexive, symmetric & transitive, it is an equivalence relation.

And Equivalence Class $[(3,4)]$ is:

$[(1,2),(2,3),(3,4),(4,5),(5,6),(6,7),(7,8),(8,9),(9,10)]$

28) $\quad y = x^x \quad —①$

$\Rightarrow \log y = x \log x.$

differentiating b/s w.r.t. 'x' we get: $\quad \dfrac{1}{y}\dfrac{dy}{dx} = \log x + x \cdot \dfrac{1}{x}$

Differentiating b/s again w.r.t. 'x':

$$\frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$$

or $$\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0 \quad \therefore \text{ Hence Proved.}$$

OR

i) f is continuous on $[0, \pi/2]$

ii) Here $f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$

$= 4\sin x \cos x\,(\sin^2 x - \cos^2 x)$

$= 2\sin 2x\,(-\cos 2x) = -\sin 4x$

i.e $f'(x) = -\sin 4x$ $\Rightarrow f$ is differentiable on $(0, \pi/2)$

Also $\left.\begin{matrix} f(0) = 1 \\ \& \ f(\pi/2) = 1 \end{matrix}\right\rangle \Rightarrow f(0) = f(\pi/2)$

All conditions of Rolle's Theorem are satisfied so $\exists$ at least one pt $c \in (0, \pi/2)$ s.t $f'(c) = 0$

$\Rightarrow -\sin 4c = 0$

or $4c = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \text{------}$

$\Rightarrow c = 0, \pm\frac{\pi}{4}, \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \text{-----}$

$\therefore c = \pi/4$

$\therefore$ Rolle's Theorem holds true for the given function with $c = \pi/4$

Ans.

29) $$\frac{dx}{dy} = \frac{x - \sqrt{xy}}{y} \quad \text{—} (1)$$

Let $x = vy$ — (2)

$\therefore \frac{dx}{dy} = v + y\frac{dv}{dy}$ (3)

Using (2) & (3) in (1) we get:

$$\cancel{v} + y\frac{dv}{dy} = \frac{vy - \sqrt{vy^2}}{y} = \cancel{v} - \sqrt{v}$$

$$\Rightarrow \frac{dv}{\sqrt{v}} = -\frac{dy}{y}$$

$$\Rightarrow \int v^{-1/2} dv = -\int \frac{1}{y} dy$$

$$\Rightarrow \frac{v^{1/2}}{1/2} = -\ln|y| + c$$

$$\Rightarrow 2\sqrt{v} = -\ln|y| + c$$

or $$2\sqrt{\frac{x}{y}} = -\ln|y| + C$$

or $$2\sqrt{\frac{x}{y}} + \log|y| = C \quad \text{Ans.}$$

30) Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}\,dx$ — (1)

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos\left(\cancel{\pi/2} - \pi/2 - x\right)}{1 + e^{-\pi/2 + \pi/2 - x}}\,dx$$

or $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^{-x}}\,dx = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+\frac{1}{e^x}}\,dx = \int_{-\pi/2}^{\pi/2} \frac{e^x \cos x}{1+e^x}\,dx$ — (2)

(1) + (2) gives :

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(e^x + \cancel{1})\cos x}{e^x \cancel{+1}}\,dx = \left[\sin x\right]_{-\pi/2}^{\pi/2}$$

or $2I = \left(1 - (-1)\right) = 2$

or $\boxed{I = 1}$ Ans

31) A → selected student is taller than 1.75 m
E_1 → selected " " a girl
E_2 → " " " a boy

Reqd Prob = $P\left(E_1/A\right) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$

$$= \frac{\frac{6\cancel{0}}{\cancel{100}} \times \frac{1}{\cancel{100}}}{\frac{6\cancel{0}}{\cancel{100}} \times \frac{1}{\cancel{100}} + \frac{4\cancel{0}}{\cancel{100}} \times \frac{4}{\cancel{100}}} = \frac{\cancel{6}\,3}{\cancel{22}\,11}$$

Ans.

32) Let factory I operate for 'x' days
& " II " " 'y'

Min $C = 12000x + 15000y$

subject to
$50x + 40y \geq 6400$ i.e $5x + 4y \geq 640$
$50x + 20y \geq 4000$ $5x + 2y \geq 400$
$30x + 40y \geq 4800$ $3x + 4y \geq 480$
$x, y \geq 0$

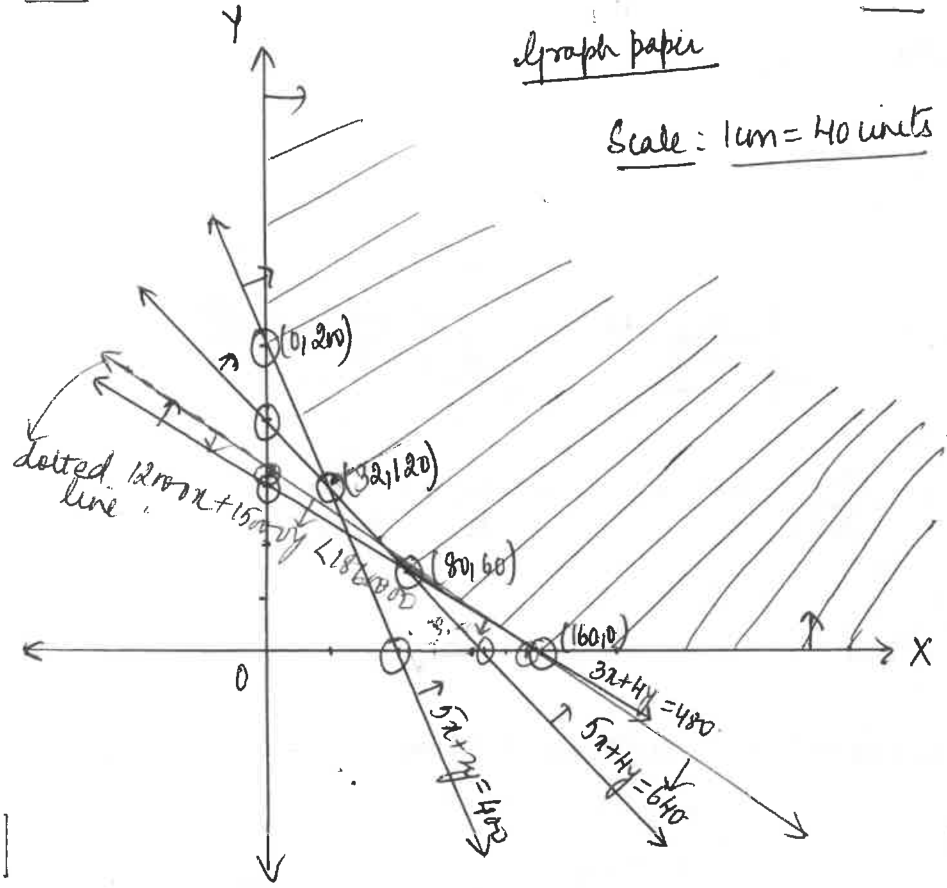
[redacted]

(80, 60)
$5x + 4y = 640$ → (128, 0), (0, 160)
(0, 200) $5x + 2y = 400$ (32, 120)
(80, 0) $3x + 4y = 480$ → (0, 120), (160, 0)

$\left(\frac{320}{7}, \frac{600}{7}\right)$

Graph paper

Scale : 1cm = 40 units



The shaded part of the graph represents the feasible region with corner points (0,200), (32,120), (80,60) & (160,0)

Here

$C = 12000x + 15000y$

$\therefore C(0,200) = 30,00,000$

$C(32,120) = 2184000$

$C(80,60) = 1860000$

$C(160,0) = 1920000$

Proposed Minimum

To decide Min^m value of C, we sketch:

$12000x + 15000 < 1860000$ on the same graph.

i.e $12x + 15y < 1860$

or $4x + 5y < 620$ ⟶ has no points common with the feasible region

$\therefore$ Minimum Value of C = 1860000 at (80,60)

i.e Z is minm when Factory I operates for 80 days & Factory II for

SECTION-D

33) Here $|A| = 2(-4+4) + 3(-6+4) + 5(3-2)$

$= -6 + 5 = -1 \neq 0 \Rightarrow A^{-1}$ exists.

Cofactor matrix $A = \begin{bmatrix} 0 & +2 & 1 \\ -1 & -9 & -5 \\ 2 & +23 & 13 \end{bmatrix}$

$\text{Adj}A = C^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$\therefore A^{-1} = \frac{\text{Adj}A}{|A|} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

The given system of eqns is

$2x - 3y + 5z = 16$.

$3x + 2y - 4z = -4$

$x + y - 2z = -3$

which can be expressed as.

$AX = B$.

$\Rightarrow X = A^{-1}B = \begin{bmatrix} 0 & 1 & +2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 16 \\ -4 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \Rightarrow x = 2,\ y = 1\ \&\ z = 3$

Check:

Putting values of x, y & z in the 1st eqn:

$2(2) - 3(1) + 5(3) = 16$

$4 - 3 + 15 = 16$

or $16 = 16$ ✓

$$\boxed{\therefore \text{Ans: } x = 2, y = 1 \ \& \ z = 3}$$

OR

$$L.H.S = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ca & cb & a^2+b^2 \end{vmatrix}$$

$R_1 \to aR_1, \ R_2 \to bR_2, \ R_3 \to cR_3$

$$L.H.S = \frac{1}{abc} \begin{vmatrix} a(b^2+c^2) & a^2b & a^2c \\ ab^2 & b(c^2+a^2) & b^2c \\ c^2a & c^2b & c(a^2+b^2) \end{vmatrix}$$

Taking a, b, c common from C_1, C_2, C_3 respectively

$$L.H.S = \frac{abc}{abc} \begin{vmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$R_1 \to R_1 - R_2 - R_3$

$$LHS = \begin{vmatrix} 0 & -2c^2 & -2b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$$\text{or } LHS = (-2) \begin{vmatrix} 0 & c^2 & b^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{vmatrix}$$

$C_2 \to b^2 C_2$, $C_3 \to c^2 C_3$

$$LHS = \frac{-2}{b^2c^2}\begin{vmatrix} 0 & b^2c^2 & c^2b^2 \\ b^2 & b^2(c^2+a^2) & c^2b^2 \\ c^2 & b^2c^2 & c^2(a^2+b^2) \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$LHS = \frac{-2}{b^2c^2}\begin{vmatrix} 0 & 0 & c^2b^2 \\ b^2 & b^2a^2 & c^2b^2 \\ c^2 & -a^2c^2 & c^2(a^2+b^2) \end{vmatrix}$$

Exp. along R_1 we get:

$$\Rightarrow LHS = \frac{-2}{b^2\cancel{c^2}}(\cancel{c^2}b^2)\begin{vmatrix} b^2 & b^2a^2 \\ c^2 & -a^2c^2 \end{vmatrix}$$

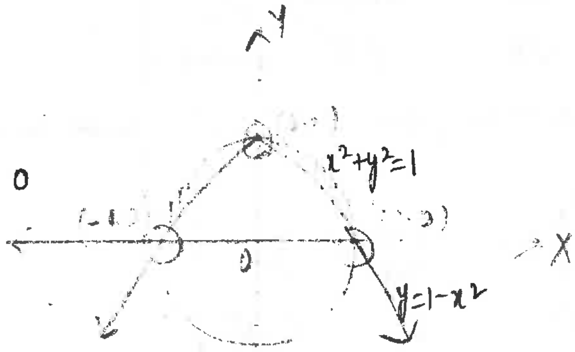
$$\Rightarrow LHS = (-2)(-2a^2b^2c^2) = 4a^2b^2c^2 = RHS$$ Hence Proved

34) $x^2+y^2=1$ —① ↳ $C(0,0)$ & $r = 1$ unit

$y = 1-x^2$
or $x^2 = 1-y$
or $x^2 = -(y-1)$ —②
↳ a parabola op ↓ $V(0,1)$

Pt. of intersection of ① & ②:

$y^2 + \cancel{1} - y = \cancel{1}$
or $y(y-1) = 0$
$\Rightarrow y = 0, 1$
$y=0 \Rightarrow x = \pm 1$; $y=1 \Rightarrow x = 0$



The shaded part of the figure represents the reqd region
& Reqd area $= A = 2\left|\int_0^1 \left(\sqrt{1-x^2} - (1-x^2)\right)dx\right|$

$$= 2\left|\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right)_0^1 - (x)_0^1 + \frac{1}{3}(x^3)_0^1\right|$$

$$= 2\left|\left(0 + \frac{\pi}{4} - 0 + 0\right) - (1-0) + \frac{1}{3}(1-0)\right|$$

$= 2\left|\frac{\pi}{4} - \frac{2}{3}\right|$

$= \left(\frac{\pi}{2} - \frac{4}{3}\right)$ sq units Ans

35) Let C be the cost of fuel

$\Rightarrow C \propto v^2$

or $C = kv^2$ — (1)

Given : $C = ₹48/hr$ when $v = 16\,km/hr$

$\Rightarrow 48 = k(16)^2 \Rightarrow k = 3/16$

$\therefore C = \frac{3}{16}v^2$

cost of fuel per hr.

Let P be the total cost when the train run a total distance of 's' km in 't' hours.

$\therefore P = \frac{3}{16}v^2 \times t + 300t$

$\Rightarrow P = \frac{3v^{\cancel{2}}}{16} \times \frac{s}{\cancel{v}} + \frac{300s}{v}$

$\therefore \frac{dP}{dv} = \frac{3s}{16} - 300s\left(\frac{1}{v^2}\right)$

$\frac{dP}{dv} = 0 \Rightarrow \frac{3\cancel{s}}{\cancel{16}} = \frac{\overset{100}{\cancel{300}}\cancel{s}}{v^2}$

$\Rightarrow v^2 = 1600$ or $v = 40\,km/hr$

$\frac{d^2P}{dv^2} = -3000\left(\frac{-2}{v^3}\right) = \frac{6000}{v^3} > 0$

$\Rightarrow$ P is min^m for $v = 40\,km/hr$.

$\therefore$ The most economical speed is $40\,km/hr$.

36) Let the C-eqn of the st. line be:

$\frac{x-3}{a} = \frac{y-0}{b} = \frac{z-1}{c}$ — (1)

(1) is ‖ to the plane $x + 2y = 0$ (2) $\Rightarrow$ (1) is ⊥ to the normal to plane (2)

$\Rightarrow a + 2b = 0$ — (3)

Similarly $3b - c = 0$ — (4)

Solving ③ & ④ we get

$$\frac{a}{-2-0} = \frac{-b}{-1-0} = \frac{c}{3-0} = \lambda \text{ (say)}$$

$\Rightarrow a = -2\lambda$
$b = \lambda$
$c = 3\lambda$

$\therefore$ C. Eq. of the line is: $\dfrac{x-3}{-2} = \dfrac{y-0}{1} = \dfrac{z-1}{3}$

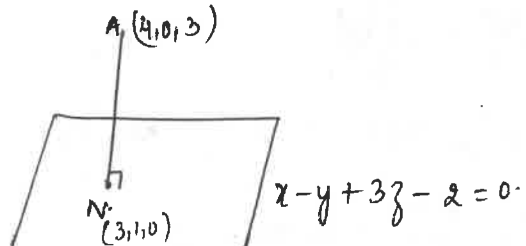
& V. eqn of the line is: $\vec{r} = 3\hat{i} + \hat{k} + \lambda(-2\hat{i} + \hat{j} + 3\hat{k})$

<u>OR</u>

d.r's of the ⊥ line : 1, −1, 3.
Plane is ⊥ to line
$\Rightarrow$ Normal to the plane is ∥ to the line
$\therefore$ Eqn of plane passing through (1, 2, 1) & ⊥ to given line is:

$$1(x-1) + (-1)(y-2) + 3(z-1) = 0$$

or $\boxed{x - y + 3z - 2 = 0.}$ Answer



Eq of AN is: $\dfrac{x-4}{1} = \dfrac{y-0}{-1} = \dfrac{z-3}{3} = \lambda \text{ (say)}$

$\therefore$ Co-ords of any gen pt on AN (say N) are:
$(\lambda+4, -\lambda, 3\lambda+3)$

But N lies on the plane $x - y + 3z - 2 = 0$.

$\therefore \lambda + 4 + \lambda + 3(3\lambda + 3) - 2 = 0$
$\Rightarrow 11\lambda + 11 = 0$
$\Rightarrow \lambda = -1$ $\therefore$ Pt N is: $(3, 1, 0) \rightarrow$ foot of ⊥ from (4, 0, 3) to the plane

& ⊥ dist $= |AN| = \sqrt{(4-3)^2 + (0-1)^2 + (3-0)^2} = \sqrt{1+1+9} = \sqrt{11}$ units Ans