Solutions - Periodic Test-3-2019-20

Class - XII - Subject - Mathematics

Section - A

1. (c) 2. (d) 3. (a) 4. $-2/5$

5. $\frac{d^2y}{dx^2}=0$ 6. 6 sq units

Section - B

7. $x\,dy = y\,dx - x^2e^x\,dx$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - xe^x$

or $\frac{dy}{dx} - \frac{y}{x} = -xe^x$ which is a linear D.E of the type $\frac{dy}{dx}+Py=Q$.

I.F $= e^{-\int \frac{1}{x}dx} = e^{-\log x} = e^{\log x^{-1}} = 1/x$

$\therefore$ Soln is: $y\left(\frac{1}{x}\right) = \int (-xe^x)\cdot\frac{1}{x}\,dx$

$\Rightarrow \frac{y}{x} = -e^x + C$

or $\boxed{y = x(C-e^x)}$ — Ans

OR

$\frac{dy}{y+1} = \frac{dx}{x-1}$

Integrating b/s we get:

$\int \frac{dy}{y+1} = \int \frac{dx}{x-1}$

$\Rightarrow \ln|y+1| = \ln|x-1| + \ln C$

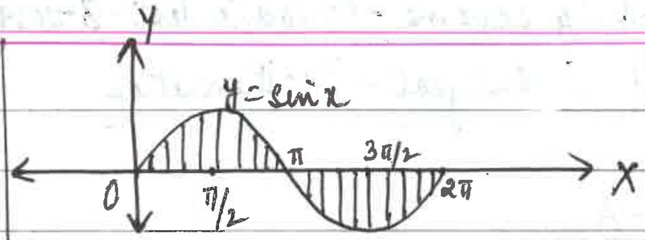
or $(y+1) = C(x-1)$ or $C = \frac{y+1}{x-1}$

When $x=1$, $y=2$. $\Rightarrow C=\infty$

$\Rightarrow x-1=0$

or $\boxed{x=1}$ Ans.

8.



The shaded part represents the reqd area

& Reqd Area $= \left|\int_0^{\pi} \sin x\,dx\right| + \left|\int_{\pi}^{2\pi} \sin x\,dx\right|$

$= \left|(-\cos x)_0^{\pi}\right| + \left|(-\cos x)_{\pi}^{2\pi}\right|$

$= \left|-(-1-1)\right| + \left|-(1-(-1))\right|$

$= 2 + 2 = 4$ sq units

Section – C

9. $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ — (1) which is a homogeneous differential equation.

Let $y = vx$ — (2)

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ — (3)

Using (2) & (3) in (1) we get:

$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2x^2v} = \frac{1+v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1+v^2-2v^2}{2v}$

or $\frac{2v}{1-v^2}dv = \frac{dx}{x}$

Integrating b/s we get:

$\int \frac{2v}{1-v^2}dv = \int \frac{dx}{x}$

Let $1-v^2 = t$

$\Rightarrow -2v\,dv = dt$

$\therefore -\int \frac{dt}{t} = \int \frac{dx}{x}$

$\Rightarrow -\ln t = \ln x + \ln c$

$\Rightarrow \ln t^{-1} = \ln(cx)$

$\Rightarrow \frac{1}{t} = cx$

or $\dfrac{1}{1-v^2} = cx$

$\Rightarrow \dfrac{1}{1-\frac{y^2}{x^2}} = cx$

or $\dfrac{x^2}{x^2-y^2} = cx$

given: when $x=2,\ y=1$

$\therefore \dfrac{4}{4-1} = 2c$

$\Rightarrow c = 2/3$

$\therefore$ Reqd soln is: $x^2 = \dfrac{2x}{3}(x^2-y^2)$

or $3x^2 - 2x^3 + 2xy^2 = 0$.

or $\boxed{x(-2x^2+3x+2y^2)=0}$ Ans.

Section D

10. $y = x^2-4x+5$ — (1) $y = x+1$ — (2)

or $y = \underbrace{x^2-4x+4}+1$

or $y-1 = (x-2)^2$ which is a parabola with vertex $(2,1)$ opening upwards.

For Pt. of intersection of (1) & (2):

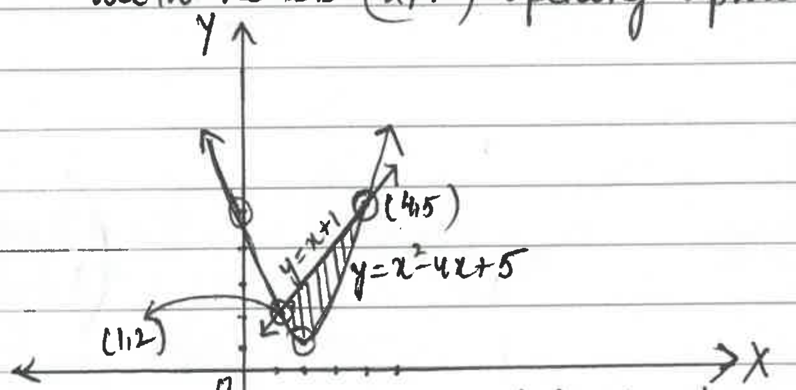
$x+1 = x^2-4x+5$

$\Rightarrow x^2-5x+4=0$

or $(x-1)(x-4)=0$

$\Rightarrow x = 1, 4$

$\therefore$ Pts of intersection of (1) & (2) are: $(1,2)$ & $(4,5)$



The shaded part represents the reqd area &

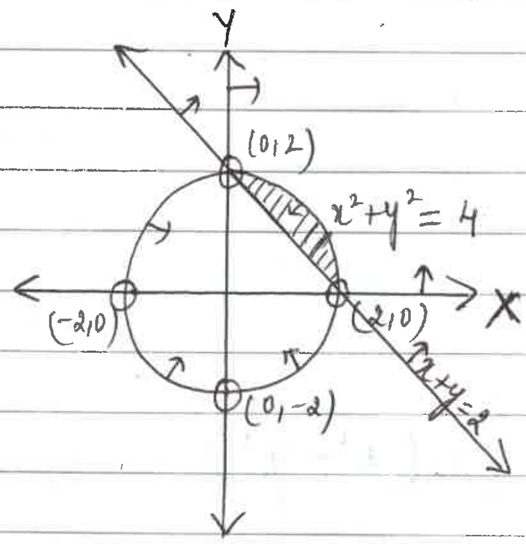
Reqd Area $= \displaystyle\int_{x=1}^{4} [(x+1)-(x^2-4x+5)]\,dx$

$= \left(\dfrac{x^2}{2}\right)_1^4 + (x)_1^4 - \left(\dfrac{x^3}{3}\right)_1^4 + 4\left(\dfrac{x^2}{2}\right)_1^4 - 5(x)_1^4$

$= \dfrac{1}{2}(16-1) + (4-1) - \dfrac{1}{3}(64-1) + 2(16-1) - 5(4-1)$

$\Rightarrow$ Reqd Area = $9/2$ sq units Ans.

OR



The shaded part represents the required area.

$$\therefore \text{Reqd Area} = \int_{x=0}^{2} \left(\sqrt{4-x^2} - (2-x)\right) dx$$

$$\Rightarrow \text{Reqd Area} = \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - 2(x)_0^2 + \left(\frac{x^2}{2}\right)_0^2$$

$$= \left[0 + 2(\sin^{-1}1 - 0)\right] - 2(2) + \frac{1}{2}(2^2)$$

$$= \cancel{2}(\pi/\cancel{2}) - 4 + 2$$

$$= (\pi - 2) \text{ sq units Ans.}$$

62 copies Xlm.