* Solutions - Periodic Test - I - class XII - 2019-20 - Sub - MATHEMATICS

Section - A

Q1 (d) , Q2 (b) , Q3 → $x = 5$, Q4 → False , Q5 → True

Q6 → (i)(a) & (ii)(c)

Section - B

Q7) $(AB' - BA')' = (AB')' - (BA')'$

$= (B')'A' - (A')'B'$

$= BA' - AB'$

$= -(AB' - BA')$

$\Rightarrow AB' - BA'$ is skew symmetric

Q8) $AA' = \frac{1}{3} \times \frac{1}{3} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix} \begin{bmatrix} 1 & 2 & x \\ 2 & 1 & 2 \\ 2 & -2 & y \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 & 0 & x+4+2y \\ 0 & 9 & 2x+2-2y \\ x+4+2y & \underset{\downarrow}{} \;\; 2x+2-2y & x^2+4+y^2 \end{bmatrix}$

Acc. to the Question :

$AA' = I$

$$\Rightarrow \frac{1}{9} \begin{bmatrix} 9 & 0 & x+4+2y \\ 0 & 9 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+y^2+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \frac{1}{9}(x+4+2y) = 0$ & $\frac{1}{9}(2x+2-2y) = 0$

$\Rightarrow \cancel{x} + 2y = -4$

& $\cancel{-x} + y = -1$

$3y = -3$

$\Rightarrow y = -1$ & $x = y - 1 = -1 - 1 = -2$

$\therefore x + y = -2 - 1 = -3$ Ans.

For one-one

Let $gof(x_1) = gof(x_2)$; $x_1, x_2 \in R$ dom

$\Rightarrow \frac{2x_1+3}{\cancel{2}} = \frac{2x_2+3}{\cancel{2}}$

$\Rightarrow 2x_1+\cancel{3} = 2x_2+\cancel{3}$

$\Rightarrow x_1 = x_2$

$\Rightarrow gof$ is one-one.

For onto

Let $y \in R$ codom

s.t. $y = gof(x)$

$\Rightarrow y = \frac{2x+3}{2}$

or $x = \frac{2y-3}{2} \in R$ dom

$\forall$ elt in the codomain R $\exists$ a pre-image in the domain R

$\Rightarrow$ gof is onto

gof is one-one onto

$\Rightarrow (gof)^{-1}$ exists & is defined as: $(gof)^{-1}: R \to R$

given by $(gof)^{-1}(y) = \frac{2y-3}{2}$.

12) $\therefore$ LHS $= \begin{vmatrix} y^2z^2 & yz & y+z \\ z^2x^2 & zx & z+x \\ x^2y^2 & xy & x+y \end{vmatrix}$

$R_1 \to xR_1$, $R_2 \to yR_2$ & $R_3 \to zR_3$

$\therefore$ LHS $= \frac{1}{xyz} \begin{vmatrix} xy^2z^2 & xyz & xy+xz \\ yz^2x^2 & xyz & yz+yx \\ zx^2y^2 & xyz & zx+zy \end{vmatrix}$

$= \frac{\cancel{xyz}}{\cancel{xyz}} \begin{vmatrix} xy^2z^2 & 1 & xy+xz \\ yz^2x^2 & 1 & yz+yx \\ zx^2y^2 & 1 & zx+zy \end{vmatrix}$ (Taking xyz common from C_2)

Taking xyz common from C_1

LHS $= xyz \begin{vmatrix} yz & 1 & xy+xz \\ zx & 1 & yz+yx \\ xy & 1 & zx+zy \end{vmatrix}$

$C_1 \to C_1 + C_3$

LHS $= xyz \begin{vmatrix} xy+yz+zx & 1 & xy+xz \\ xy+yz+zx & 1 & yz+yx \\ xy+yz+zx & 1 & zx+zy \end{vmatrix}$

Taking $(xy+yz+zx)$ common from C_1

$= (xy+yz+zx)(xyz) \begin{vmatrix} 1 & 1 & xy+xz \\ 1 & 1 & yz+yx \\ 1 & 1 & zx+zy \end{vmatrix}$

$= xyz(xy+yz+zx) \times 0$ ($\because C_1$ & C_2 are identical)

$= 0 =$ RHS Hence Proved.

8) OR $A^2 = 8A + kI$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix} \Rightarrow 1 = 8+k \text{ or } k = -7$$

$\therefore A^2 = 8A - 7I$

$\Rightarrow A^2A^{-1} = 8AA^{-1} - 7IA^{-1} \Rightarrow A = 8I - 7A^{-1}$

or $A^{-1} = \dfrac{8I - A}{7} = \dfrac{1}{7}\left\{\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\right\} = \dfrac{1}{7}\begin{bmatrix} 7 & 0 \\ 1 & 1 \end{bmatrix}$

A^{-1} Ans.

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9) $aRb \Leftrightarrow \sin^2 a + \cos^2 b = 1$; $a, b \in$ set of Real Nos.

(i) Reflexivity: aRa $\because \sin^2 a + \cos^2 a = 1 \ \forall a \in$ set of real no.'s.

$\Rightarrow R$ is reflexive

(ii) Transitivity: Let aRb & bRc ; $a, b, c \in$ set of real no.'s.

$\Leftrightarrow \sin^2 a + \cos^2 b = 1$ & $\sin^2 b + \cos^2 c = 1$

$\Leftrightarrow \sin^2 a + \cancel{1} - \sin^2 b = \cancel{1}$ & $\sin^2 b + \cos^2 c = 1$

$\Leftrightarrow \sin^2 a - \sin^2 b = 0$ — (1) & $\sin^2 b + \cos^2 c = 1$ — (2)

Adding (1) & (2) we get :

$\Leftrightarrow \sin^2 a + \cos^2 c = 1$

$\Rightarrow aRc \Rightarrow R$ is transitive.

$\therefore$ Given relation is both reflexive & transitive.

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10) $\tan^{-1}\left(\dfrac{1/4 + 2/9}{1 - \frac{1}{4}\times\frac{2}{9}}\right) = \dfrac{1}{2}\cos^{-1}x$

$\Rightarrow \tan^{-1}\left(\dfrac{\frac{9+8}{36}}{34/36}\right) = \dfrac{1}{2}\cos^{-1}x$

$\Rightarrow 2\tan^{-1}\left(\dfrac{1}{2}\right) = \cos^{-1}x$

$\Rightarrow \cos^{-1}\left(\dfrac{1-(1/2)^2}{1+(\frac{1}{2})^2}\right) = \cos^{-1}x$

$\Rightarrow \dfrac{1 - 1/4}{1 + 1/4} = \cos^{-1}x$ or $x = 3/5$

$\therefore \cos(\cos^{-1} + 2\sin^{-1}x)$

$= \cos(\pi/2 + \sin^{-1}x)$

$= -\sin(\sin^{-1}x) = -x = -3/5$ Ans.

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11) $f: R \to R$ is given by $f(x) = \dfrac{2x-1}{2}$

& $g: R \to R$ is given by $g(x) = x+2$

$\therefore gof: R \to R$ & is given by $gof(x) = g(f(x)) = g\left(\dfrac{2x-1}{2}\right)$

$= \dfrac{2x-1}{2} + 2 = \dfrac{2x+3}{2}$.

12) OR. $LHS = \begin{vmatrix} 0 & xyz & x-z \\ y-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix}$

$C_1 \to C_1 - C_3$

$LHS = \begin{vmatrix} z-x & xyz & x-z \\ z-x & 0 & y-z \\ z-x & z-y & 0 \end{vmatrix} = (z-x)\begin{vmatrix} 1 & xyz & x-z \\ 1 & 0 & y-z \\ 1 & z-y & 0 \end{vmatrix}$

~~LHS~~ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$LHS = (z-x)\begin{vmatrix} 1 & xyz & x-z \\ 0 & -xyz & y-x \\ 0 & z-y-xyz & z-x \end{vmatrix}$

Expanding along C_1

$LHS = (z-x)\begin{vmatrix} -xyz & y-x \\ z-y-xyz & z-x \end{vmatrix}$

$= (z-x)\{(-xyz)(z-x) - (y-x)(z-y-xyz)\}$

$= (z-x)\{-xyz^2 + \cancel{x^2yz} - yz + y^2 + xy^2z + xz - xy \cancel{- x^2yz}\}$

$= (z-x)\{xyz(y-z) + y(y-z) + x(z-y)\}$

$= (z-x)(y-z)(xyz + y - x) = RHS$ Hence Proved.

Section D

(13) Here $|A| = \begin{vmatrix} 1 & 1 & 3 \\ 1 & -1 & 1 \\ -1 & -1 & -2 \end{vmatrix} = 1(2+1) - 1(-2+1) + 3(-1-1) = 3 + 1 - 6 = -2 \neq 0 \Rightarrow A^{-1}$ exists.

Cofactor matrix $C = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 1 & 0 \\ 4 & 2 & -2 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{Adj A}{|A|} = \frac{C^T}{|A|} = -\frac{1}{2}\begin{bmatrix} 3 & -1 & 4 \\ 1 & 1 & 2 \\ -2 & 0 & -2 \end{bmatrix}$

The given system of eqns can be expressed as $A^T X = B \Rightarrow X = (A^T)^{-1}B$
$\Rightarrow X = (A^{-1})^T B$

or $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} 3 & 1 & -2 \\ -1 & 1 & 0 \\ 4 & 2 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = -\frac{1}{2}\begin{bmatrix} -4 \\ -2 \\ -4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} \Rightarrow x = 2, y = 1, z = 2$ Ans.